Gravity in gauge mediation

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ABSTRACT: We investigate O'Raifeartaigh-type models for F-term supersymmetry breaking in gauge mediation scenarios in the presence of gravity. It is pointed out that the vacuum structure of those models is such that in metastable vacua gravity mediation contribution to scalar masses is always suppressed to the level below 1 percent, almost sufficient for avoiding FCNC problem. Close to that limit, gravitino mass can be in the range 10–100 GeV, opening several interesting possibilities for gauge mediation models, including Giudice-Masiero mechanism for μ and $B\mu$ generation. Gravity sector can include stabilized moduli.

Contents

1. Introduction

Gauge mediation of supersymmetry breaking to the Standard Model sector [1–11] is an interesting alternative to gravity mediation. A considerable attention has recently been paid to gauge mediation models with very heavy messengers and the gravitino mass in the GeV range [12–14], as a phenomenologically interesting possibility. Gravitino in that mass range is a viable dark matter candidate, consistently with a high reheating temperature needed for leptogenesis. [15–24]. On the other hand, there has been a revival of interest in O'Raifeartaigh-type mechanism of F-term supersymmetry breaking [25] as the source of supersymmetry breaking in gauge mediation scenarios, see, e.g. [26–37]. This is partly motivated by the wide recognition of the role of metastable supersymmetry breaking vacua in models with R-symmetry broken spontaneously and/or explicitly by a small parameter [38]. With explicit models of supersymmetry breaking in gauge mediation scenarios at

hand and with special interest in high scale supersymmetry breaking vacua, it is important to investigate in detail the role of gravity in such scenarios.

Any physically acceptable model of spontaneous symmetry breaking needs gravity for eliminating massless goldstino by the super-Higgs mechanism. But gravity may also be important for determination of the supersymmetry breaking vacua. A simple example is a model discussed in [26] with the superpotential containing a linear term of a gauge singlet chiral superfield X responsible for supersymmetry breakdown. The messengers Q and q, transmitting the supersymmetry breakdown to the visible sector, transform as $\mathbf{5}$ and $\bar{\mathbf{5}}$ of SU(5) and are coupled to the field X. The superpotential reads $W = FX - \tilde{\lambda}XQq$ and the form of the Kähler potential,

$$K = \bar{X}X - \frac{(\bar{X}X)^2}{\tilde{\Lambda}^2} + \bar{q}q + \bar{Q}Q \,, \tag{1.1}$$

takes into account the loop corrections representing the logarithmic divergence in the effective potential coming from the effects of the massive fields in the O'Raifeartaigh model which have been integrated out. The sign of the second term in the Kähler potential is negative here. For $\tilde{\lambda} = 0$ supersymmetry is broken by $F_X \neq 0$ and X is stabilized at 0. Supersymmetry is however restored by turning on the coupling λ. The supersymmetric global minimum is at $X = 0$ and $Q = q = \sqrt{F/\tilde{\lambda}}$. Coupling the model to gravity changes the vacuum structure. Supersymmetric vacuum is still present as a global minimum (with shifted values of the fields) but in addition a local (metastable) minimum with broken supersymmetry appears, with vanishing vevs for the messengers fields.

A constant c is added to the superpotential to cancel the cosmological constant at the metastable vacuum. Another fact worth mentioning is that, with gravity, after decoupling the messengers ($\tilde{\lambda} = 0$) the supersymmetric vacuum disappears (as in the case without gravity) but the minimum is for X different from zero. In the above discussion, the role of gravity is linked to the negative sign of the second term in the Kähler potential, which follows from the O'Raifeartaigh model. However, more general models of a similar type can give positive sign for that term and X can be stabilized away from the origin, with broken supersymmetry [39], with or without messengers even in the limit $M_P \to \infty$. One can expect that the role of gravity is then more subtle.

In both cases, the models without messengers can be used for supersymmetry breaking and F-term vacuum uplifting in the gravity mediation scenarios [40 – 44].

The purpose of this note is to investigate a certain broad class of O'Raifeartaigh-type models (ORT models) [31] as the mechanism for supersymmetry breaking in gauge mediation scenarios, coupled to gravity. The ORT models for supersymmetry breaking may contain only SM singlets and then for gauge mediation of supersymmetry breaking they must be supplemented with the messenger sector. The ORT models may themselves contain fields charged under SM gauge symmetries, participating in the dynamics of supersymmetry breaking and simultaneously transmitting it to the SM. Finally, the supersymmetry breaking sector may obey the SU(5) symmetry or SU(5) symmetry may be explicitly broken by the masses and couplings. The gravity sector may or may not contain additional fields (moduli).

We investigate the vacuum structure of the globally supersymmetric models and after coupling them to gravity. We find that in the latter case, metastable local minima with broken supersymmetry exist only for the values of the supersymmetry breaking field X stabilized below certain upper bound of order $10^{-3}M_P$. In addition, at the tree level, there always exists a stable Polonyi-type minimum with $X \approx M_P$ but we are interested only in the vacua with X stabilized at values such that quantum corrections from the messenger and O'Raifeartaigh sectors are under perturbative control.

This result has interesting implications for the possible importance of gravity mediation relative to gauge mediation (see also [45]) in models where gravity is coupled to the ORT models of supersymmetry breaking, in the presence of charged under SM gauge group messengers. After coupling to gravity, in metastable vacua gauge mediation remains always the dominant mechanism. Another version of the same conclusion is that, in moduli stabilization models with F-term uplifting in metastable vacua, adding messengers changes gravity mediation into dominantly gauge mediation. At the same time, our results show that gauge mediation with messenger masses up to 10^{15} GeV and gravitino mass up to 100 GeV is a viable possibility. We investigate some phenomenological consequences of such scenarios. One question is whether the range of values of the μ and $B\mu$ parameters consistent with the electroweak breaking can be compatible with Giudice-Masiero mechanism [46]. The idea of employing gravity to solve the μ problem of gauge mediation bears some resemblance to the motivation of [12], though the tools we use and the models we build are quite different.

We begin in section 2 by reviewing globally symmetric ORT models and classifying them according to the existence or nonexistence of supersymmetry breaking vacuum with $X \neq 0$. In section 3 we discuss the role of gravity. We show that coupling to gravity has important implications for the accessible range of values for stabilized X. We also consider the gravity sector with moduli fields and discuss the moduli stabilization in this context. We also review scenarios that, potentially, could lead to the dominance of gravity mediation in our present general set-up. In section 4 we turn our attention to ORT models in which the messenger sector does not exhibit the unified SU(5) symmetry. Such models have been advocated in [31] as a possible remedy to the fine-tuning problem of the MSSM. We study some simple but viable examples of these models and determine their properties. We then proceed to section 5 where we discuss the phenomenological implications of the models studied in sections 2 and 4 in the presence of the upper bound on the value of X found in section 3.1. Finally, section 6 contains our conclusions.

2. Properties of metastable supersymmetry breaking vacua in generalized O'Raifeartaigh models

In this section we discuss globally supersymmetric ORT models. A simple reference model is the original O'Raifeartaigh model itself [25], with its effective form mentioned in the Introduction. We consider a single field X with $R(X) = 2$ and with a linear term in the superpotential, and a sector consisting of N pairs of fields ϕ_i, $\tilde{\phi}_i$ belonging to n_ϕ-

dimensional conjugate representations of a gauge group.[1] We assume that at tree level these fields have canonical Kähler potential and that their interactions are described by the most general superpotential consistent with R symmetry:

$$W = FX + \sum_{i=1}^{N} \sum_{j=1}^{N} \tilde{\phi}_i (m_{ij} + \lambda_{ij} X) \phi_j \,, \tag{2.1}$$

where the mass parameter m_{ij} can be nonzero if and only if $R(\tilde{\phi}_i) + R(\phi_j) = 2$, and the coupling λ_{ij} can be nonzero if and only if $R(\tilde{\phi}_i) + R(\phi_j) = 0$. Without loss of generality, we can order the fields so that the $R(\phi_j)$ do not increase and $R(\tilde{\phi}_j)$ do not decrease with j. Interactions of X with $\tilde{\phi}_i$ and ϕ_j induce an effective potential for X. In the leading order in the parameter $F/\bar{m}^2$, where $\bar{m}$ is a representative mass scale of m_i, the presence of the effective potential can be accounted for by introducing a correction to the Kähler potential [38]:

$$\delta K = -\frac{1}{16\pi^2} \mathrm{Tr} \left[\mathcal{M}^\dagger \mathcal{M} \ln \left(\frac{\mathcal{M}^\dagger \mathcal{M}}{Q^2} \right) \right] \,, \tag{2.2}$$

where Q is the cutoff of the theory (we take it equal to M_P) and we denote by $\mathcal{M}_{ij} = m_{ij} + \lambda_{ij} X$ the X-dependent mass matrix for $\tilde{\phi}_j$, ϕ_i. It is known that gaugino masses generated at one loop are proportional to $F \partial_X \ln \det \mathcal{M}$. Thus, in order to generate gaugino masses at one loop, $\det \mathcal{M}$ must depend on X. However, in [39], it is shown that for $\det m \neq 0$ R-symmetry implies that $\det \mathcal{M} = \det m$. Therefore, in order to generate gaugino masses at one loop, $\mathcal{M}$ must be singular in the limit $X \to 0$ (i.e. matrix m must have at least one zero eigenvalue) [31]. One possibility of constructing such a model consists in choosing R-charges so that $\mathcal{M}$ can be split into a singular and nonsingular part as:

$$\mathcal{M}(X) = \begin{pmatrix} \tilde{\lambda} X & 0 \\ 0 & \mathcal{M}^{\mathrm{ns}}(X) \end{pmatrix} \,, \tag{2.3}$$

where the matrix $\mathcal{M}^{\mathrm{ns}}(X)$ is the maximal submatrix of $\mathcal{M}$ such that $\det \mathcal{M}^{\mathrm{ns}}$ does not depend on X. We now denote by $\tilde{\lambda}$ the submatrix of λ which has not been included in $\mathcal{M}^{\mathrm{ns}}$.

A few comments are in order here. Firstly, the form (2.3) is by no means necessary for constructing a successful model of supersymmetry breaking and its mediation to the visible sector, as shown explicitly in [31] and as we show later in appendix B with our examples (v) and (vi). However, the form (2.3) is a perfectly admissible possibility, dependent only upon the assignment of the R-charges and we thereby find it interesting to explore the structure of such models. The two sectors of the $\tilde{\phi}_i$ and ϕ_j fields distinguished by (2.3) need not belong to the same gauge representation. Indeed, for generating soft supersymmetry breaking masses of the MSSM fields only the sector coupled to $\tilde{\lambda} X$ has to transform nontrivially under the gauge group (we shall call it the messenger sector), while it is possible that the sector coupling to $\mathcal{M}^{\mathrm{ns}}$ consists only of gauge singlets. However, the nonsinglets of the latter

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[1]For self-conjugate representations, we may identify the two sets of fields, simultaneously dividing n_ϕ by 2 in our subsequent formulae. In the case of singlets we may also have to impose a parity symmetry under which these fields are odd, in order to distinguish them from the field X.

sector (which we shall call the O'Raifeartaigh sector), if present, can give contributions to the masses of the MSSM scalars. These contributions are of the order of $\sim F\langle X\rangle/(m^{\text{ns}})^2$, where m^{ns} is a typical mass scale of $\mathcal{M}^{\text{ns}}$, so they can (but do not have to) be much smaller than the contribution of the first sector which are of the order of $\sim F/\langle X\rangle$.

Finally, up to the presence of the messenger sector which becomes massless in the limit $X \to 0$, the models characterized by (2.3) are the Type I models of [31], and we shall henceforth refer to them in this manner.

We shall now discuss the vacuum structure of that class of globally supersymmetric models under the additional assumption that messengers becoming massless in the limit $X \to 0$ give negligible contribution to the Kähler potential (2.2). Thus, we assume that the block $\mathcal{M}^{\text{ns}}$ in (2.3) gives the dominant contribution to (2.2). A global unitary transformation of the fields of the same R-charge allows writing $m_{ij} = m_i\delta_{ij}$ with real and positive m_i. The case of degenerate mass parameters can be treated as a simple limit of a nondegenerate spectrum. We can then expand the correction (2.2) to the Kähler potential in powers of $|X|$ as:

$$\delta K = -\frac{n_\phi\bar{m}^2}{16\pi^2}\sum_{\ell=0}^{\infty} f_{2\ell}\cdot\left(\frac{\bar{\lambda}|X|}{\bar{m}}\right)^{2\ell}. \tag{2.4}$$

In (2.4) we have extracted the overall dependence on the representative mass scale $\bar{m}$ and coupling strength $\bar{\lambda}$, so that the dimensionless functions $f_{2\ell}$ depend only on ratios $\lambda_{ij}/\bar{\lambda}$ and $\rho_i \equiv m_i/\bar{m}$. Note that the scale Λ appearing in eq. (1.1) is now given (for $f_4 > 0$) by $\Lambda = 4\pi\bar{m}/(\bar{\lambda}^2 n_\phi^{1/2}|f_4|^{1/2})$ and it can be significantly larger than the scale $\bar{m}$. In terms of the coefficients f_i, the scalar potential

$$V(\tilde{\phi}_I, \phi_J, X) = (K_{\bar{X}X})^{-1}|F + \tilde{\lambda}_{ij}\tilde{\phi}_i\phi_j|^2 + |X|^2\left(\tilde{\lambda}_{ij}\tilde{\lambda}_{ik}^*\phi_j\phi_k + \tilde{\lambda}_{ij}\tilde{\lambda}_{kj}^*\tilde{\phi}_i\tilde{\phi}_k^*\right) \tag{2.5}$$

reads for $\tilde{\phi}_i = \phi_j = 0$:

$$V(|X|) = F^2\left[1 + \frac{n_\phi\bar{\lambda}^2}{16\pi^2}\left(4f_4\left(\frac{\bar{\lambda}|X|}{\bar{m}}\right)^2 + 9f_6\left(\frac{\bar{\lambda}|X|}{\bar{m}}\right)^4 + \cdots\right)\right]\ldots \tag{2.6}$$

In this expansion we denoted by ... terms of higher order in $|X|$ as well as contributions from the messenger sector. We also suppressed the effects of the $|X|^0$ and $|X|^2$ in (2.4). The former amounts to overall rescaling of the superpotential and the latter restores canonical normalization of X by rescaling $X \to (1+(n_\phi\bar{\lambda}^2 f_2)(16\pi^2))X$; both are corrections of higher order in powers of $\bar{\lambda}^2/(16\pi^2)$.

Obviously, the vacuum structure in the limit $\tilde{\lambda} \to 0$ depends on the signs of the coefficients f_4 and f_6. For $f_4 > 0$ there is a minimum for $X = 0$ with broken supersymmetry. We see the correspondence to the discussion in the Introduction. Turning on the couplings $\tilde{\lambda}$ we recover supersymmetric minimum at $X = 0$. For $f_4 < 0$ the extremum is for $X \neq 0$ and its character depends on the sign of f_6. Also the value of X in the extremum depends of f_6. Turning on the couplings $\tilde{\lambda}$ we get a global supersymmetric minimum at $X = 0$, too, but for $f_6 > 0$ the metastable minimum at $X \neq 0$ and with broken supersymmetry is still present. The presence of a supersymmetry breaking minimum with $X \neq 0$ may be endangered by

turning on couplings $\tilde{\lambda}$ in three ways: a large contribution of $\tilde{\lambda}$ to the scalar potential or a development of tachyonic masses in the X and/or messenger sector. None of these situation occurs if, parametrically, $F/|X|^2 < |\tilde{\lambda}_{ij}| \ll \bar{\lambda}^2|X|/\bar{m}$. Since the sign of f_4 distinguishes between the two significantly different possibilities, it is interesting to determine f_4 for a generic O'Raifeartaigh sector. Generally, the functions $f_{2\ell}$ depend in a rather complicated way on the original model parameters. A general method for finding them and explicit results for a somewhat simplified class of models, in which one can write $\lambda_{ij} = \bar{\lambda}q_i e^{\iota\varphi_i}\delta_{i+1,j}$, are presented in appendix A. Using these results, we can calculate f_4 and, if necessary, f_6 in general Type I models, as shown with a few simple examples in appendix B.

We have so far assumed that the model is described by the reducible matrix (2.3). We have briefly analyzed the necessary conditions for the messenger sector not to affect the position of a metastable supersymmetry breaking minimum (if it exists). Here we would like to extend the class of models under consideration by still keeping the reducible form of the matrix $\mathcal{M}$ but replacing the submatrix $\tilde{\lambda}X$ in (2.3) by

$$\mathcal{M}^{\rm s} = \begin{pmatrix} \tilde{\lambda}_1 X & \tilde{m}_1 & & & \\ & \tilde{\lambda}_2 X & \tilde{m}_2 & & \\ & & \ddots & \ddots & \\ & & & \tilde{\lambda}_{\tilde{N}-1} X & \tilde{m}_{\tilde{N}-1} \\ & & & & \tilde{\lambda}_{\tilde{N}} X \end{pmatrix} \tag{2.7}$$

Writing this expression we assumed that in the messenger sector all the fields ϕ_i have different R-charges and that all the fields $\tilde{\phi}_j$ have different R-charges. The generalization consists in adding explicit mass terms in the messenger sector, but in such a way that in the limit $X \to 0$ the matrix $\mathcal{M}^{\rm s}$ has at least one zero eigenvalue and the gaugino masses can be generated. In the language of [31], our model is now a hybrid of a Type I and Type II model which is classified there as a Type III model. From now on we shall assume that all the mass terms $\tilde{m}_i$ are nonzero, otherwise $\mathcal{M}^{\rm s}$ can be split in two independent blocks whose contributions to the effective potential add up. We would like to check if the present form of the messenger sector can result in developing a supersymmetry breaking minimum with $X \neq 0$ despite $f_4 > 0$. As before we can extract a representative coupling strength $\bar{\lambda}'$ and a representative mass scale $\bar{m}'$ and define coupling and mass ratios as $q_i' \equiv \tilde{\lambda}_i/\bar{\lambda}$ and $\rho_i' \equiv \tilde{m}_i/\bar{m}$. Around $X = 0$ the eigenvalues of $\mathcal{M}^{\rm s\dagger}\mathcal{M}^{\rm s}$ can then be expanded as $\bar{m}^2(\rho_i')^2 + \mathrm{O}(\bar{\lambda}'X/\bar{m}')$ and $f_0(\bar{\lambda}|X|/\bar{m})^{2\tilde{N}}$, where $f_0 = \prod_{i=1}^{\tilde{N}}(q_i')^2/\prod_{j=1}^{\tilde{N}-1}(\rho_j')^2$. The effective Kähler potential (2.2) can then be expanded as:

$$\delta K = -\frac{n_\phi \bar{m}^2}{16\pi^2}\left(\ldots + f_4 \cdot \left(\frac{\bar{\lambda}|X|}{\bar{m}}\right)^4 + 2\tilde{N}f_0\left(\frac{\bar{\lambda}|X|}{\bar{m}}\right)^{2N}\ln\left(\frac{\bar{\lambda}|X|}{\bar{m}}\right) + \cdots\right). \tag{2.8}$$

In (2.8) we included the lowest-order nontrivial monomial (recall that a $|X|^2$ in δK only rescales X) and the only term proportional to $\ln|X|$. For simplicity we also assumed that the messenger and the O'Raifeartaigh sector are made of the same representations. We also set $\bar{\lambda}' = \bar{\lambda}$ and $\bar{m}' = \bar{m}$ assuming that this choice leads to a reliable expansion of δK. Now f_4 is a sum of two contributions: one from the O'Raifeartaigh sector and the second

coming from the messenger sector. The latter can also be calculated from our formulae in appendix A with q_j replaced by q'_{j+1} for $j = 1, \ldots, \tilde{N} - 1$ and with additional contributions from $q'_{\tilde{N}} = q'_1$, $\rho'_{\tilde{N}} = 0$ and $\rho'_{\tilde{N}+1} = \rho_1$. The logarithmic term in (2.8) generates an additional contribution $8\tilde{N}^3 f_0 (\bar{\lambda}|X|/\bar{m})^{(2\tilde{N}-2)} \ln(\bar{\lambda}|X|/\bar{m})$ to the expansion of the effective potential. For small values of $|X|$ the negative logarithmic term may compensate a positive value of f_4 to generate a metastable minimum with $X \neq 0$. If $f_4 < 0$ then both the quartic and the logarithmic term in (2.8) pull X away from zero and one should consider higher-order terms to find the position of the minimum. A particular case of this scenario is a possibility that the O'Raifeartaigh sector is absent and a metastable supersymmetry breaking minimum arises only through interactions of the messenger sector (Type II models of [31]). Some examples of models in which the messenger sector can affect the position of the supersymmetry breaking minimum are discussed in appendix B.

We have so far assumed that the interactions stabilizing X at the supersymmetry breaking minimum obey a unified SU(5) symmetry (which includes the case of the O'Raifeartaigh sector consisting of gauge singlets). It has been argued that going beyond this assumption may lead to interesting phenomenological consequences (e.g. higgsino (N)LSP, light gluinos and/or stops, small fine-tuning) in both gravity mediated [47] and gauge mediated [31, 48] scenarios of supersymmetry breaking. Such models (with gauge mediation) will be discussed in section 4.

In conclusion, we have discussed in this section globally supersymmetric ORT models. It is clear that the vacuum structure of such models depends on their details. Both options are open: they may or may not have metastable supersymmetry breaking vacua, with $X \neq 0$.

3. Supergravity vacua of generalized O'Raifeartaigh models

3.1 Constraints on the scale of supersymmetry breaking

In this section we couple an ORT model to gravity, as an illustration of gravity effects on such models. We have checked that those effects are generic and our results apply to all models considered here. They are of two kinds: one can be anticipated on the basis of the results of [26] – for ORT models with $f_4 > 0$, i.e. with no minimum with broken supersymmetry and $X \neq 0$, gravity effects lead nevertheless to a metastable minimum of that kind. The other gravity effect is universal — for models with and without metastable minimum with $X \neq 0$ in the global limit, inclusion of gravity leads to the vacuum structure such that there is an upper bound on the values of X in the metastable vacua, $X \ll M_P$. In addition, for the same values of parameters, there always exist a tree-level Polonyi-like minimum with broken supersymmetry and with $X \approx M_P$. Clearly, those effects have interesting physical implications for the gauge mediation models, to be discussed in the next section.

In supergravity, the scalar potential for X is given by:

$$V(X) = e^{K/M_P^2} \left[\left(\partial_X W + (\partial_X K/M_P^2) W \right) \left(\partial_{\bar{X}} \bar{W} + (\partial_{\bar{X}} K/M_P^2) \bar{W} \right) - 3W\bar{W}/M_P^4 \right] , \quad (3.1)$$

where we take K to be the sum of the canonical Kähler potential and the one-loop correction (2.2), and we assume that the superpotential W is (2.1) plus a constant term c necessary to cancel approximately the cosmological constant.

For $X > 0$, the scalar potential can be written in a form similar to (2.6):

$$V(X) = F^2 \left[1 - \frac{3c^2}{F^2 M_P^2} - \frac{4c}{F M_P^2} X + \right.$$
$$\left. + \frac{n_\phi \bar{\lambda}^2}{16\pi^2} \left(1 + \frac{2c}{F M_P^2} X \right) \left(4f_4 \left(\frac{\bar{\lambda} X}{\bar{m}} \right)^2 + 9f_6 \left(\frac{\bar{\lambda} X}{\bar{m}} \right)^4 + \cdots \right) + \cdots \right], \quad (3.2)$$

where the first and second ... denote term of higher orders in X and M_P^{-1}, respectively. The basic difference between the scalar potential in supergravity and in the globally supersymmetric case (2.6) is the presence of odd powers of X [26]. In particular, if the minimum at $X = 0$ exist in the global limit, i.e. $f_4 > 0$, then the linear term always destabilizes it, shifting it to a nonzero value. As an illustrative example we consider a model admitting both types of vacua, depending on the values if its parameters, namely, that described in appendix B, in Example (iii) with $r = 1$. Upon identification $\bar{m} = m/2$, $\bar{\lambda} = \lambda$ and $n_\phi = 1/2$, this model is equivalent to the model with singlet fields and the superpotential:

$$W = m\phi_1 \phi_3 + \frac{R}{2} m\phi_2^2 + \lambda \phi_1 \phi_2 X + FX + c. \quad (3.3)$$

A model described by (3.3) has been extensively studied in the global limit in [39]. Here we couple it to gravity and we again assume that the messenger sector does not affect the position of the local supersymmetry breaking minimum. When $c = F M_P / \sqrt{3}$ the effective potential (3.2) vanishes in the limit $|X| \to 0$. Only small corrections to this relation are necessary to make the cosmological constant vanish at the supersymmetry breaking local minimum of the potential with $X \neq 0$ and we shall use this approximate relation from now on.

Functions f_4 and f_6, defined in eq. (2.4), have the following form:

$$f_4 = -\frac{1 + 2R^2 - 3R^4 + R^2(R^2 + 3)\ln R^2}{(R^2 - 1)^3} \quad (3.4)$$

$$f_6 = \frac{1 + 27R^2 - 9R^4 - 19R^6 + 6R^2(R^4 + 5R^2 + 2)\ln R^2}{3(R^2 - 1)^5}. \quad (3.5)$$

As shown in [39], the function f_4 is positive for $R < 2.11$ and negative otherwise (see also figure 11 in appendix B). The function f_6 is positive for $R > 1/2$, thereby ensuring (in the global limit) the existence of a metastable supersymmetry breaking minimum whenever $f_4 < 0$.

If the approximation in the effective potential shown in (3.2) is sufficiently accurate to determine the local supersymmetry breaking minimum with $X \neq 0$, we can envision three main classes of solutions depending on the sign and size of f_4. For $f_4 < 0$ and $f_6 > 0$ we recover the minimum previously discussed in the context of globally supersymmetric model:

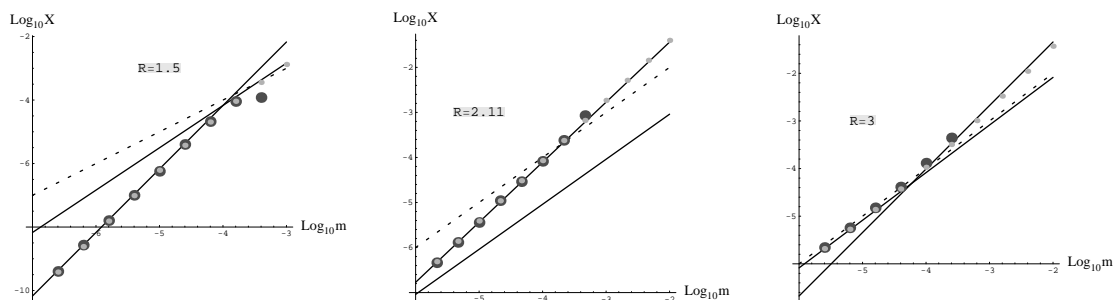$$X^2 = \frac{8|f_4|}{9f_6} \frac{\bar{m}^2}{\bar{\lambda}^2}. \quad (3.6)$$

Figure 1: Positions of the minimum of the full supergravity potential for $R = 1.5$, $R = 2.11$ and $R = 3$ shown as large dark dots (full calculation), small light dots (δK truncated at $O(|X|^6)$) and solid lines (approximate solutions described in the text) for $\lambda = 1$. The dashed lines show $\lambda X / m = 1$.

For $f_4 > 0$ the position of the supersymmetry breaking minimum is determined by a balance between terms linear and quadratic in X [26]. The solution reads then:

$$X = \frac{1}{2\sqrt{3}} \frac{\Lambda^2}{M_P} , \tag{3.7}$$

where, as before, Λ appearing in (1.1) is given by $\Lambda = 4\pi\bar{m}/(\bar{\lambda}^2 n_\phi^{1/2} |f_4|^{1/2})$. Finally, we may have $f_4 \approx 0$, leading to a dominance of the quartic term over the quadratic one. We find then:

$$X^3 = \frac{16\pi^2}{9\sqrt{3} n_\phi f_6} \frac{\bar{m}^4}{\bar{\lambda}^6 M_P} . \tag{3.8}$$

Note that including the supergravity corrections to the effective potential is crucial for the existence of solutions (3.7) and (3.8), for which $\langle X \rangle$ is proportional to negative powers of M_P. All the three solutions (3.6)–(3.8) break R-symmetry [49]: in (3.6) R-symmetry is broken spontaneously, whereas the form of (3.7) and (3.8) shows that explicit soft R-symmetry breaking (the constant term c in the superpotential) is transmitted to $\langle X \rangle$ through gravitational interactions. In figure 1, we present results of numerical analysis for $R = 3$, $R = 2.11$ and $R = 1.5$, basically corresponding to the solutions (3.6), (3.8) and (3.7), respectively, in terms of the original parameter m in (3.3). We conservatively assumed a rather large value $\lambda = 1$. Large dark dots represent the position of the minima calculated in the full supergravity potential with δK defined in (2.2) and with the cosmological constant vanishing at the minimum. Small light gray dots correspond to minima calculated in the full supergravity potential, but with δK truncated at $O(|X|^6)$ terms. Pairs of solid lines show the solutions (3.6) and (3.8) for $R = 3$ and $R = 2.11$, and the solutions (3.7) and (3.8) for $R = 1.5$. The dashed lines show $\lambda X / m = 1$, i.e. the value of X for which the expansion in (2.4) is expected to break down. Our numerical analysis shows that all the three solutions can be realized in the simple model (3.3), depending on the value of the mass ratio R. We can also see that for increasing values of m (and thus $\langle X \rangle$) the quartic term

in (3.2) becomes more important and for given R solutions (3.6) and (3.7) can be replaced by (3.8). However, unless $|f_4| \ll f_6$, this regime occurs rather close to the $\lambda X/m = 1$.

We note that the local minimum disappears for mass scales of the O'Raifeartaigh sector slightly smaller than $10^{-3} M_P$ This can be understood by noticing that the solution (3.8), which is a good approximation for sufficiently large $\bar{m}$, can be rewritten as:

$$\left(\frac{\bar{\lambda} X}{\bar{m}}\right)^3 = \frac{32\pi^2}{9\sqrt{3} f_6} \frac{\bar{m}}{\bar{\lambda}^3 M_P^3} \tag{3.9}$$

If the left-hand side of (3.8) exceeds unity, our perturbative expansion (2.4) breaks down and one should not expect the minimum to persist. It also follows from (3.9) that the maximal scale $\bar{m}$ for which there exists a local minimum with $X \neq 0$ scales as $\bar{\lambda}^3$, which, upon substitution to (3.8) shows that the corresponding value of X scales as $\bar{\lambda}^2$. These observations are confirmed by our numerical analysis. Hence our example with $\bar{\lambda} = 1$ is a conservative choice indeed.

We repeated the numerical analysis described above for several randomly generated models with the O'Raifeartaigh sector consisting from $N = 4$ to $N = 10$ pairs of fields (for the details of the randomization procedure, see Example (iv) in appendix B) and checked if the metastable minimum disappears for large $\bar{m}$. We found that this is the case, indeed; in each example we had $X/M_P < O(10^{-3})$, which shows that the results for the particular example discussed in this section can be applied to a broad class of ORT models, including those with broken SU(5) symmetry, to be discussed in section 4. This bound can, however, be made weaker by introducing a messenger and/or O'Raifeartaigh sector with a very large number of fields, so that the one-loop corrections can compete more efficiently with M_P-suppressed corrections.

We also note that although the three possibilities discussed above form an exhaustive list for the simple model we have considered, in more general situations one can have $f_4 < 0$ and $f_6 < 0$, and to determine the position of the local minimum of the potential (2.6) or (3.2) – if it exists — one has to consider terms of higher powers of X. Since in this case one obtains a solution that survives in the limit $M_P \to \infty$, we believe that it has similar properties to (3.6).

The numerical analysis supports the conclusion that the supergravity corrections provide an upper bound on the values of the mass scale $\bar{m}$ of the O'Raifeartaigh sector for which the metastable supersymmetry breaking minima exist. In the particular example analyzed here, this bound is $\bar{m} \lesssim 10^{-3} M_P$, which corresponds to $\Lambda \lesssim 10^{-2} M_P$.

3.2 Gauge mediation in the presence of moduli

In this subsection we would like to extend our discussion to gauge mediation in the presence of moduli or, vice versa, to the impact of messengers on moduli stabilization. The presence of moduli is a generic future of gravitational sectors rooted in extra-dimensional models. In fact, ORT models, with SM singlets only, have been used as the source of supersymmetry breaking and the uplifting potential in supergravity models with moduli, stabilized by supersymmetric dynamics involving fluxes and nonperturbative superpotentials [40, 42–44]. In simple models discussed in the literature, one assumes that all moduli except one (let

us call it T) are stabilized by fluxes and T is stabilized by a nonperturbative superpotential. Its Kähler potential reads $K' = -3\ln(T + \bar{T})$ and a general superpotential $W^c(T)$ involves a constant term and nonpertubative term, such as that of the KKLT model [50], $W^c(T) = W_0 + Ae^{-aT}$, or the KL model $W^c(T) = W_0 + Ae^{-aT} - Be^{-bT}$ [51]. The minimization of the supergravity potential for the modulus in the absence of the O'Raifeartaigh sector amounts to solving $D_T W = 0$, where $D_T W = W_T - (3/(T + \bar{T}))(W/M_P^2)$, i.e. the minimum with T stabilized at T_0 is supersymmetric and in AdS.

It is rather clear (see e.g. [42]) that the shift δT induced by the O'Raifeartaigh sector is small, provided the supersymmetry breaking parameter F in eq. (2.1) is $F \ll M_P^2$ (we assume here that the parameters in the superpotential (2.1) do not depend on the values of the moduli). Furthermore, the induced supersymmetry breaking in the modulus sector is small. $F_T \ll F_X$. Hence, in such simple models the presence of the modulus sector stabilized at the tree level does not interfere with supersymmetry breaking in the ORT models, even if gravitationally induced like in [26] or in models with $f_4 > 0$ discussed in section 2. The moduli sector replaces the constant c in the potential (3.3), to fine-tune the cosmological constant to zero.

It is worth checking the constraints on the moduli sector in the presence of messengers in the ORT models coupled to gravity with stabilized moduli. The scalar potential now reads (for real parameters A, W_0 and for $X > 0$):

$$V(X,T) = \frac{F^2 M_P^3}{(T + \bar{T})^3}\left[1 - \frac{(T + \bar{T})^2 |W_T^c|^2}{3F^2 M_P^2} - \cdots - \frac{4W^c}{FM_P^2}X + \right.$$
$$\left. + \frac{n_\phi \bar{\lambda}^2}{16\pi^2}\left(1 + \frac{2W^c}{FM_P^2}X\right)\left(4f_4\left(\frac{\bar{\lambda}X}{\bar{m}}\right)^2 + 9f_6\left(\frac{\bar{\lambda}X}{\bar{m}}\right)^4 + \cdots\right) + \cdots\right], \quad (3.10)$$

where we assumed the validity of supergravity approximation $aT \gg 1$ and denoted by $\ldots$ terms subleading in powers of aT. The new element of the present setup is the constraint following from the cancellation of the cosmological constant, correlating the value of the F parameter in the superpotential (2.1) with the value of T in the presence of the dominantly gauge mediated soft terms.

In the KKLT model uplifted by the O'Raifeartaigh sector [44], the condition of the cosmological constant cancellation in the limit $X \to 0$, following from (3.10), gives

$$\frac{2}{\sqrt{3}M_P}AaTe^{-aT} = F, \quad (3.11)$$

(again corrections from the fact that the actual minimum is located at $0 < \langle X \rangle \ll M_P$ are negligible). Also, since both the supersymmetric masses squared and supersymmetry breaking mass squared splitting within multiplets are rescaled by a factor $e^{K'}(T + \bar{T})^{-3}$ both the gravitino mass $m_{3/2}$ and the soft masses have to be multiplied by a factor $e^{K'/2} = (T + \bar{T})^{-3/2}$. With this correction, using equation (3.11) and relation $m_{\text{gaugino}} = (\alpha/4\pi)(F/\langle X \rangle)$ (cf. eq. (5.1) in section 5) and noting that the coupling to the modulus does not alter the value of X in the minimum we obtain the following lower
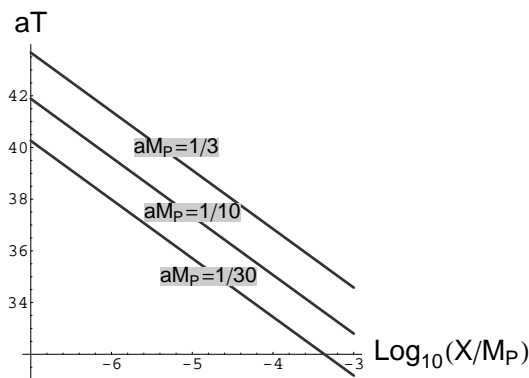
Figure 2: Values of aT following from relation (3.12) as a function of $\langle X \rangle / M_P$ for three values of $aM_P = 1/3, 1/10, 1/30$.

bound on the stabilized modulus T:

$$(aM_P)^{3/2} \frac{e^{-aT}}{\sqrt{aT}} = 10^{-14} \left(\frac{m_{\text{gaugino}}}{100 \, \text{GeV}} \right) \left(\frac{0.04}{\alpha} \right) \left(\frac{M_P^3}{A} \right) \frac{\langle X \rangle}{M_P}. \tag{3.12}$$

For the reference values given above, the value of the modulus, T, following from this relation is shown in figure 2 as a function of of $\langle X \rangle / M_P$ for three values of $aM_P = 1/3, 1/10, 1/30$. Changing the right-hand side of (3.12) by a factor of a few does not result in any significant changes of the value T. These rather large values of T_{min} reflect, in part, the smallness of the ratio (5.2) in ORT models requiring the modulus vev larger by $\sim \frac{1}{a} \ln(m_{\text{gaugino}}/m_{3/2})$ with respect to usual scenarios with gravity mediation [44]. The mass scale of X depends on which supergravity solution is realized: for (3.6) we can estimate: $m_X \sim O(1) \, \text{TeV} \, (m_{\text{gaugino}}/100 \, \text{GeV})(0.04/\alpha)(\bar{\lambda}/1)$, while for (3.7) we find $m_X \sim O(1) \, \text{TeV} \, (m_{\text{gaugino}}/100 \, \text{GeV})(0.04/\alpha) \sqrt{10^3 \langle X \rangle / M_P}$. The mass scale of the modulus is $m_T \sim (aT)m_{3/2} \sim (2/3)M_P(A/M_P^3)(aM_P)^{3/2}(aT)^{1/2}e^{-aT}$; a requirement that so estimated $m_T > 100 \, \text{GeV}$ can be translated to $aT < 34, 35.5, 37.5$ for $aM_P = 1/30, 1/10, 1/3$, respectively. Depending on cosmological history such a light, only gravitationally coupled scalar is prone to introduce the well-known cosmological moduli problem.

We note that one can attempt to loosen the correspondence between the parameter F and the gravitino mass, thereby relaxing the bounds discussed above. An example of such a strategy consists in coupling the messengers to a fields that plays only a secondary role in supersymmetry breaking, which suppresses the gauge mediated contributions to soft masses (see also [52]). The minimal possibility of this type utilizes $F_T \ll F_X$ and employs a superpotential coupling $\tilde{\lambda} e^{-bT} Qq$ instead of $\tilde{\lambda} X Qq$. This leads to the dominance of gravity mediation, introducing potentially large FCNC effects, or, if gravity mediation can be suppressed by means of sequestering, the dominance of the anomaly mediation giving

negative masses squared to sleptons, though the latter problem may potentially be cured by $\sim (\alpha/4\pi)(F_X\langle X\rangle/\bar{m}^2)$ contributions to the scalar soft masses, mediated by the gauge interactions of nonsinglets in the O'Raifeartaigh sector. Instead of coupling messengers only to moduli one can also extend the model by adding another singlet X' with a superpotential $F'X'$ with $F' \gg F$. The field X' can be stabilized at a supersymmetry breaking minimum with an O'Raifeartaigh sector of its own or by one shared with the field X. Both scenarios go beyond the scope of this work and we shall leave them for a future study.

Various mass relations discussed above result from the fact that with $aT \gg 1$, the modulus sector has practically one mass scale tightly connected to F through cancellation of the cosmological constant. More general models, e.g. the KL model [51] obtained by adding another exponential term to the superpotential, allow for a complete separation of the modulus and messenger sector and the above conclusions following from (3.11) no longer hold.

4. Models with broken SU(5) symmetry

We have so far discussed models whose messenger sectors obey the unified SU(5) gauge symmetry. Motivated by [31], where breaking of SU(5) symmetry allowed constructing models of gauge mediation with a low messenger scale and a small μ parameter, we would like to extend this discussion to models with an arbitrary messenger scale. Here we review the properties of such simple models, concentrating on the predictions which depend on the structure of these models rather than on the actual model parameters. This discussion is then continued in section 5, where we calculate the low energy spectra of such models and discuss their consequences.

We begin by recalling the general formulae for the soft masses in gauge mediated models. Using the Giudice-Rattazzi method of extracting the supersymmetry breaking effects from wave function renormalization [53] we find the one loop gaugino masses [31]:

$$M_r = \frac{g_r^2}{16\pi^2} F \, \partial_X \ln \det \mathcal{M} \tag{4.1}$$

and the two-loop scalar masses:

$$m_{\tilde{f}}^2 = \sum_{r=1}^{3} (m_{\tilde{f}}^{(r)})^2, \qquad \text{with} \qquad (m_{\tilde{f}}^{(r)})^2 = \frac{g_r^4}{128\pi^4} C_{\tilde{f}}^{(r)} F \, \partial_{\bar{X}} \partial_X \left[\sum_{i=1}^{N} \ln^2 \left(\frac{\mathcal{M}_i^2}{\Lambda_{\rm UV}^2} \right) \right], \quad (4.2)$$

where r runs over the Standard Model gauge groups, $C_{\tilde{f}}^{(r)}$ are the appropriate Casimir invariants and $\mathcal{M}_i^2$ are the eigenvalues of the mass matrix $\mathcal{M}^\dagger \mathcal{M}$. Equations (4.1) and (4.2) are valid at the scale at which we integrate out the messenger fields,[2] below this scale a usual renormalization group analysis is required. Since the Casimir invariants are the same above and below the messenger threshold, no A-terms are generated at the messenger scale. Equations (4.1) and (4.2) also assume universal interactions within each messenger multiplet ($\mathcal{M}_i^2$ do not depend on r); if this assumption is violated, e.g. when doublets

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[2]Of course, the messenger masses can be split; corrections to (4.2) arising from messenger mass hierarchies are briefly discussed in appendix C.
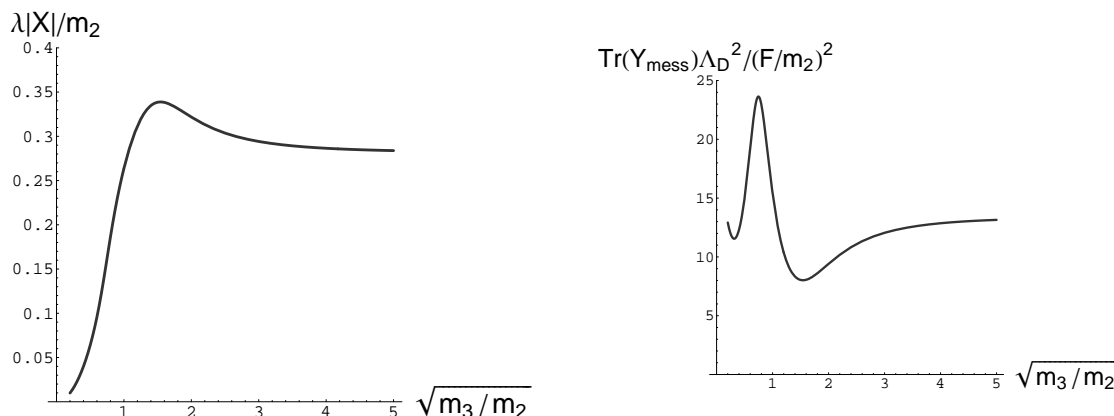
Figure 3: The location X of the supersymmetry breaking minimum and the parameter $\mathrm{Tr}[Y_{\mathrm{mess}}\Lambda_D]$ describing the one-loop contributions to scalar masses as functions of the ratio of the mass parameters in the doublet and triplet sectors, m_2 and m_3, for the model defined in (4.5).

of SU(2) and triplets of SU(3) forming a **5** representation of SU(5) have different mass matrices, we should replace $\mathcal{M}_i^2$ by their appropriate components.

In SU(5)-symmetric models with N_{mess} pairs of messengers in $\mathbf{5} + \bar{\mathbf{5}}$ representations and with supersymmetric masses proportional to $\langle X \rangle$, it follows from (4.1) and (4.2) that the soft gaugino and scalar masses obey:

$$(m_{\tilde{f}}^{(r)})^2 = \frac{2C_{\tilde{f}}^{(r)}}{N_{\mathrm{mess}}} M_r^2 \,. \tag{4.3}$$

In models with more complicated messenger interactions, in particular in those with broken SU(5) symmetry, it is convenient to define, by analogy, an *effective messenger number* [31]:

$$N_{\mathrm{eff},r} = \frac{2C_{\tilde{f}}^{(r)} M_r^2}{(m_{\tilde{f}}^{(r)})^2} \,, \tag{4.4}$$

where M_r and $(m_{\tilde{f}}^{(r)})^2$ are given by (4.1) and (4.2), respectively. Of course, in this definition we only take these scalars for which $(m_{\tilde{f}}^{(r)})^2 \neq 0$; the definition is independent of the specific choice of $\tilde{f}$. With messengers residing in $\mathbf{5} + \bar{\mathbf{5}}$ pairs (with possibly different interactions of the doublet and triplet components), the effective messenger numbers obey $5/N_{\mathrm{eff},1} = 3/N_{\mathrm{eff},2} + 2/N_{\mathrm{eff},3}$.

It has long been known that breaking the SU(5) symmetry generically leads to one-loop D-term contributions to the masses of scalars, proportional to their hypercharge and thus not positive definite [9], unless the messenger sector obeys so-called *messenger parity* [54]. The absence of one-loop contributions to scalar masses is thus a nontrivial constraint on

the models. We illustrate this by studying this issue in the simplest version of the model put forward in an early version of [31], which employs three pairs of $\mathbf{5} + \bar{\mathbf{5}}$ fields whose interactions are described by the following superpotential:

$$W = FX + \lambda X(\tilde{\phi}_1 \phi_1 + \tilde{\phi}_2 \phi_2) + \lambda' \tilde{\phi}_3 \phi_3 X + m_A \tilde{\phi}_1 \phi_2 + \delta m_A \tilde{\phi}_2 \phi_3 \,, \qquad (4.5)$$

where m_2 and δm_2 (m_3 and δm_3) are the mass parameters for the doublets (triplets) residing in $\mathbf{5} + \bar{\mathbf{5}}$. Following [31], we take $\lambda' = \lambda/10$, $\delta m_2 = m_2/10$ and $\delta m_3 = 0$. We then solve numerically for the minimum of the effective potential and we calculate the one-loop contributions to the scalar masses, which in the leading order in F is given by [55]:

$$(m_{\tilde{f}}^2)_{1-\text{loop}} = \frac{\alpha_1}{4\pi} Y_{\tilde{f}} \text{Tr}[Y_{\text{mess}} (\Lambda_D)^2] \,. \qquad (4.6)$$

In this formula, the trace runs over all the Standard Model states forming the messenger multiplet and the parameter $(\Lambda_D^R)^2$ for the states in the SM representation R (SU(2) doublets and SU(3) triplets) is given by:

$$(\Lambda_D^R)^2 = \frac{1}{2} \sum_{i,j} \frac{|F_{ij}^{(R)}|^2 - |F_{ji}^{(R)}|^2}{(\mathcal{M}_i^{(R)})^2} g\left(\frac{(\mathcal{M}_j^{(R)})^2}{(\mathcal{M}_i^{(R)})^2} \right) \,, \qquad (4.7)$$

where $(\mathcal{M}_i^{(R)})^2$ are the eigenvalues of $\mathcal{M}^\dagger \mathcal{M}$ for different representations R and $F_{ij}^{(R)}$ are the supersymmetry breaking masses of the fields $\tilde{\phi}_i$ and ϕ_j in the scalar potential written in the basis in which the supersymmetric mass matrix $\mathcal{M}$ is diagonal. The scalar potential for the components of the messenger fields in representation R of the SM gauge group then reads $V = \sum_i (\mathcal{M}_i^{(R)})^2 (\phi_i^{(R)*} \phi_i^{(R)} + \tilde{\phi}_i^{(R)*} \tilde{\phi}_i^{(R)}) + \sum_{i,j} (F_{ij}^{(R)} \tilde{\phi}_i^{(R)} \phi_j^{(R)} + \text{h.c.})$. The function g is given by $g(x) = 2/(1-x) + (1+x)/(1-x)^2 \ln x$. In figure 3, we show, as a function of $\sqrt{m_3/m_2}$, the results for $\lambda |X|/m_2$ at the minimum and for $\text{Tr}[Y_{\text{mess}} (\Lambda_D)^2]$ normalized to $(F/m_2)^2$ which represents a typical mass scale (squared) in usual expressions for gauge mediation. From this we can estimate that the ratio of the two-loop contributions to one-loop ones is $\sim (1/(4\pi))(\alpha_r^2 \alpha_1)(1/10)$ which is an unavoidably small number. Since these one-loop contributions are dominant but not positive definite for right-handed sleptons, their presence is phenomenologically unacceptable.

4.1 Models with $1 \approx N_{\text{eff},2} < N_{\text{eff},3}$

In this and the following section, we shall construct models with the messenger and/or the O'Raifeartaigh sectors breaking the unified SU(5). We assume that it is more natural for the mass parameters than for the dimensionless couplings to break the unified gauge symmetry, as the former can arise from the vevs of the Standard Model singlets in nontrivial SU(5) representations. A well known example of this idea is found in various realization of the doublet-triplet splitting in the electroweak higgs sector of the unified theories, which amounts to giving large masses to the triplets which potentially mediate proton decay while leaving the electroweak higgs doublets massless at this stage.

The simple model of Example (v) in appendix B is equipped with messenger parity for $\lambda_1 = \lambda_2$, but allows for different mass parameters for the doublet and triplet components
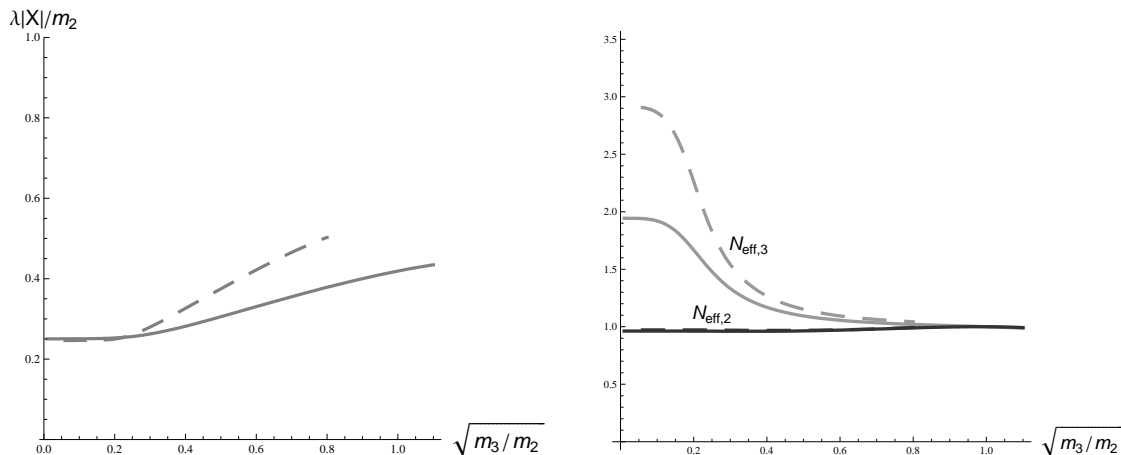
Figure 4: The location X of the supersymmetry breaking minimum and the effective number of messengers in the doublet and triplet sectors, $N_{\text{eff},2}$ and $N_{\text{eff},3}$, as functions of the ratio of the mass parameters in the doublet and triplet sectors, m_2 and m_3, for the models considered in section 4.1 with $N = 4$ (solid lines) and $N = 6$ (dashed lines).

of the messenger multiplets. However, we find that with $N = 2$ pairs of messengers, as well as in the obvious $N = 3$ generalization of this model, the difference between $N_{\text{eff},2}$ and $N_{\text{eff},3}$ is negligible for ratios of the mass parameters for doublets and triplets ranging from $1/30$ to 30, so the doublet-triplet splitting in the messenger sector is not transmitted to the visible sector. Hence the simplest model employs $N = 4$ pairs of messengers with the mass matrices for doublets and triplets are, respectively,

$$\mathcal{M}^{(2)} = \begin{pmatrix} \lambda X & m_2 & 0 & 0 \\ 0 & \lambda X & m_2 & 0 \\ 0 & 0 & \lambda X & m_2 \\ 0 & 0 & 0 & \lambda X \end{pmatrix}, \qquad \mathcal{M}^{(3)} = \begin{pmatrix} \lambda X & m_2 & 0 & 0 \\ 0 & \lambda X & m_3 & 0 \\ 0 & 0 & \lambda X & m_2 \\ 0 & 0 & 0 & \lambda X \end{pmatrix}. \qquad (4.8)$$

The value of X at the minimum of the effective potential and the effective messenger numbers are shown in figure 4. The values of the effective messenger numbers can be understood in the following way. For $m_3 \ll m_2$, the mass matrix of the triplets is effectively split into two 2×2 matrices studied in Example (v), while the mass matrix of the doublets remains irreducible. As found in [31], the mass matrix of the form of $\mathcal{M}^{(2)}$ ($\mathcal{M}^{(3)}$ with $m_3 \to 0$) provides a supersymmetry breaking metastable minimum at $\lambda|X|/m_2 = 0.45$ (0.25), and our numerical result interpolates between these two values. It also follows from [31] that for $\lambda|X|/m_2 < 1$ the effective messenger number is approximately equal to the number of irreducible sectors of the mass matrix; hence for $m_3 \ll m_2$ we have $N_{\text{eff},2} \sim 1$ and $N_{\text{eff},3} \sim 2$, whereas for $m_3 \sim m_2$ we obtain $N_{\text{eff},2} \approx N_{\text{eff},3} \sim 1$.

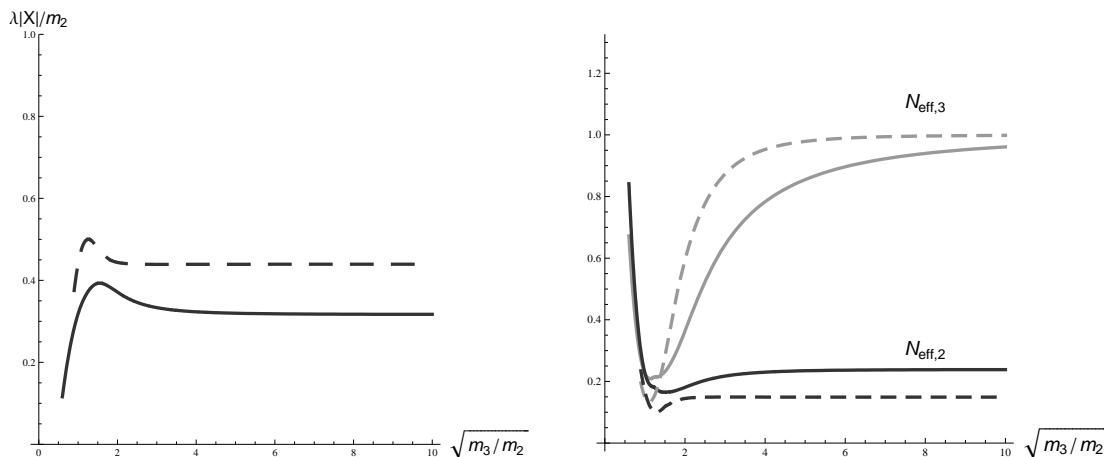By increasing the number of messengers to $N = 6$, we can engineer a larger splitting

Figure 5: The location X of the supersymmetry breaking minimum and the effective number of messengers in the doublet and triplet sectors, $N_{\text{eff},2}$ and $N_{\text{eff},3}$, as functions of the ratio of the couplings in the doublet and triplet sectors, λ_2 and λ_3, for the models considered in section 4.2. Solid and dashed lines correspond to models described by the interactions (4.10) and (4.11), respectively.

between $N_{\text{eff},2}$ and $N_{\text{eff},3}$. Indeed, with the mass matrices:

$$
\mathcal{M}^{(\mathbf{2},\mathbf{3})} = \begin{pmatrix} \lambda X & m_2 & 0 & 0 & 0 & 0 \\ 0 & \lambda X & m_{2,3} & 0 & 0 & 0 \\ 0 & 0 & \lambda X & m_2 & 0 & 0 \\ 0 & 0 & 0 & \lambda X & m_{2,3} & 0 \\ 0 & 0 & 0 & 0 & \lambda X & m_2 \\ 0 & 0 & 0 & 0 & 0 & \lambda X \end{pmatrix}
$$

we find solutions with $N_{\text{eff},2} \sim 1$ and $N_{\text{eff},3} \sim 3$; the results are also shown in figure 4.

4.2 Models with $N_{\text{eff},2} < N_{\text{eff},3} \approx 1$

The model described in Example (iii) in appendix B with $\tilde{\rho} = 1$ is invariant under the messenger parity acting as:

$$
\phi_{1,2,3} \to \tilde{\phi}^*_{3,2,1} \,, \tilde{\phi}_{1,2,3} \to \phi^*_{3,2,1} \,. \tag{4.9}
$$

We can therefore safely assume that the fields $\tilde{\phi}_i$ and ϕ_j belong to nontrivial representations of the Standard Model gauge groups and they can act as messengers of the supersymmetry breaking without obeying the unified SU(5) symmetry. Recall that we still need messenger fields which become massless in the limit $X \to 0$ in order to generate the gaugino masses at one loop. Hence the simplest model built along these lines employs the following mass matrices for doublets and triplets:

$$
\mathcal{M}^{(\mathbf{2})} = \begin{pmatrix} \lambda' X & 0 & 0 & 0 \\ 0 & m'_2 & \lambda X & 0 \\ 0 & 0 & m_2 & \lambda X \\ 0 & 0 & 0 & m'_2 \end{pmatrix}, \qquad \mathcal{M}^{(\mathbf{3})} = \begin{pmatrix} \lambda' X & 0 & 0 & 0 \\ 0 & m'_3 & \lambda X & 0 \\ 0 & 0 & m_3 & \lambda X \\ 0 & 0 & 0 & m'_3 \end{pmatrix}. \tag{4.10}
$$

To ensure the gauge coupling unification, we assume that the determinants of $\mathcal{M}^{(\mathbf{2})}$ and $\mathcal{M}^{(\mathbf{3})}$ are equal, which implies $(m_2')^2 m_2 = (m_3')^2 m_3$. Interactions of the form (4.10) stabilize X at a nonzero minimum for sufficiently large hierarchies of the mass parameters [39] and we take $m_2 = 10 m_2'$. Since the overall mass scale can be factored out in (4.10), we are then left with only one mass ratio m_3/m_2 on which the properties of the model depend. We take $\lambda' = \lambda/1000$ to allow for a separation between the value of $\langle X \rangle$ and the mass of the lightest messenger (though the results for $N_{\text{eff},r}$ would not change if $\lambda' = 1/10$). In figure 5, we plot the position of the minimum as well as the effective numbers of the messengers $N_{\text{eff},2}$ and $N_{\text{eff},3}$ in this model. We see that the asymptotic values of the messenger numbers are 0.24 and 1. We also see that $N_{\text{eff},2}$ does not reach its asymptotic value for $\lambda|X|/m_2 \to \infty$ (calculated in [31] and shown in figure 14 in appendix C) since the position of the minimum corresponds to $\lambda|X|/m_2 \sim 1/3$. By increasing the number of messengers, with explicit mass parameters, we can further change the ratio of $N_{\text{eff},3}/N_{\text{eff},2}$. For example, adding another messenger pair so that the messenger interactions are now given by:

$$
\mathcal{M}^{(\mathbf{2,3})} = \begin{pmatrix} \lambda'X & 0 & 0 & 0 & 0 \\ 0 & m_{2,3}' & \lambda X & 0 & 0 \\ 0 & 0 & m_{2,3} & \lambda X & 0 \\ 0 & 0 & 0 & m_{2,3} & \lambda X \\ 0 & 0 & 0 & 0 & m_{2,3}' \end{pmatrix}, \tag{4.11}
$$

again with $\det \mathcal{M}^{(\mathbf{2})} = \det \mathcal{M}^{(\mathbf{3})}$ and $m_2 = 10 m_2'$, leads to results shown in figure 5 with asymptotic values of the effective messenger numbers equal 0.15 and 1. By increasing the number of messengers with masses proportional to $\langle X \rangle$, we can increase all $N_{\text{eff},r}$, e.g. adding one pair of such messengers to the model defined by (4.11) gives $N_{\text{eff},2} \approx 0.7$ and $N_{\text{eff},3} \approx 2$.

All models have a stable supersymmetric vacuum. Phenomenologically consistent models with broken SU(5) symmetry can be constructed, with different effective doublet and triplet messenger numbers. With sufficiently large mass or coupling hierarchies between the doublet and triplet sectors, these effective messenger numbers depend only on the R-charges of the messenger fields, rather than on their masses and couplings. Embedding these models in supergravity still brings about an upper bound on X discussed in section 3.1, though due to a large number of fields over which the trace runs in the effective Kähler potential (2.2), one-loop corrections compete more efficiently with the supergravity corrections. For example, in the model given by (4.11) we obtain $X \lesssim 10^{-2} M_P$.

5. Physical implications

Gravity coupled to the messenger sector via ORT supersymmetry breaking sector raises the question about the role of gravity mediation versus gauge mediation of supersymmetry breaking in generating the soft supersymmetry breaking mass terms in the MSSM. The gravity mediation contribution to the scalar masses is of the order of the gravitino mass $m_{3/2} = F_X/(\sqrt{3}M_P)$, where $F_X = F$. The gauge mediated contribution to the soft scalar

and gaugino masses are (up to the (effective) messenger number and $O(1)$ coefficients) given by the scale:

$$m_{\text{gaugino}} = \frac{\alpha}{4\pi} \frac{\lambda_{\text{mess}} F_X}{\mathcal{M}_{\text{mess}}}, \tag{5.1}$$

where $\mathcal{M}_{\text{mess}}$ is the average mass of those messengers which give significant contributions to gaugino masses, the effective messenger coupling λ_{mess} is defined as $\mathcal{M}_{\text{mess}}/\langle X \rangle$, $\lambda_{\text{mess}} F_X$ is the mass squared splitting in the messenger supermultiplets and α is an appropriate coupling constant. With the simplest possible messenger sector consisting of one pair of fields in the $\mathbf{5} + \bar{\mathbf{5}}$ representations of the unified gauge symmetry group SU(5), we identify $\lambda_{\text{mess}} = \tilde{\lambda}$, but we would like to keep our discussion general enough to include all the situations discussed in the previous sections, in particular, models with large mass splittings within the messenger sector. For example, in models discussed in section 4.2 λ_{mess} can be identified with λ', ranging from 10^{-3} to 10^{-1}, even when all other universal couplings $\lambda.$ are close to 1. The main result of section 3.1 is that in the simple ORT models coupled to gravity there exists an upper bound on the value of X of order of $10^{-3} M_P$ in the metastable supersymmetry breaking minimum. An immediate conclusion is then that in metastable vacua

$$\frac{m_{3/2}}{m_{\text{gaugino}}} = \frac{4\pi}{\alpha\sqrt{3}} \frac{\langle X \rangle}{M_P} < \mathrm{O}(10^{-1}), \tag{5.2}$$

where in the last step we used conservative values of $\alpha = 0.04$ and $\langle X \rangle = 10^{-3} M_P$. Given the conservative nature of these numbers, we infer that in gauge mediation models with ORT sector responsible for F-term supersymmetry breaking coupled to gravity, in metastable vacua gravity mediation contribution to squark mass squared is naturally suppressed to at least O(1%) level, almost sufficient for avoiding FCNC problem in the squark sector [14]. The tunneling rate from the metastable vacuum at $X \ll M_P$ towards the tree-level Polonyi vacuum at $X \sim M_P$ is very small, and, in the order-of-magnitude approximation, see [56], is given by

$$\Gamma = \frac{1}{\tau_D} \approx 0.1 (\bar{m}/M_P)^4 M_P e^{-30 \frac{\bar{m}^3 M_P}{F^2}}, \tag{5.3}$$

where $\bar{m}$ is the average mass in the O'Raifeartaigh sector. This corresponds to a lifetime τ_D much longer than the age of the Universe.

From eq. (5.2) we conclude that for $m_{\text{gaugino}} \approx 100(1000)\,\text{GeV}$ the gravitino mass $m_{3/2} < \mathrm{O}(10, 100\,\text{GeV})$. (Similar values for the gravitino mass have been discussed in [13, 14]; here we show that this is a generic upper bound for gravitino mass in gauge mediation models coupled to gravity.)

The Giudice-Masiero mechanism of generating the μ-term in the effective low-energy superpotential relies on the presence in the high-energy Kähler potential of an interaction term of the form

$$\delta K = \frac{1}{2} \frac{X^\dagger}{M_P} H_u H_d + \text{h.c.}. \tag{5.4}$$

With the help of the well known formulae, see [57], one obtains this way

$$|\mu| = \left| m_{3/2} \frac{X^\dagger}{M_P} - \frac{F^{\bar{X}}}{M_P} \right| = m_{3/2} \left| \frac{X^\dagger}{M_P} - \sqrt{3} \right|, \tag{5.5}$$

	A	B	C	D	E	F	G
$N_{\text{eff},2}$	1	4	7	1	1	0.24	0.15
$N_{\text{eff},3}$	1	4	7	2	3	1	1

Table 1: Parametrization of some interesting spectra of conventional models of gauge mediation ($N_{\text{eff},2} = N_{\text{eff},3}$) and of models described in sections 4.1 and 4.2.
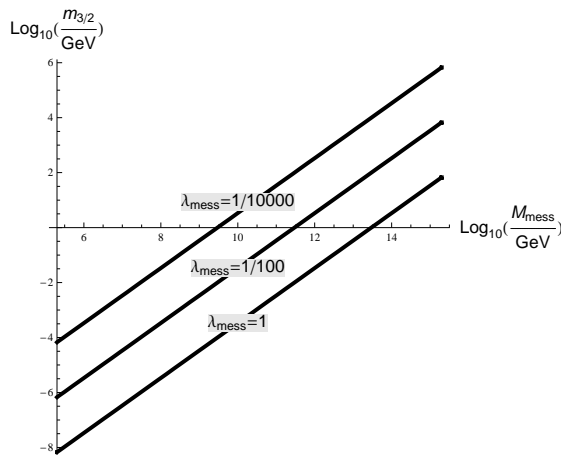

Figure 6: The relation (5.7) between $\mathcal{M}_{\text{mess}}$ and $m_{3/2}$ for different values of λ_{mess} and for $F \partial_X \ln \det \mathcal{M} = 140\,\text{TeV}$.

where the cancellation of the cosmological constant is assumed, and

$$B = \pm m_{3/2}\,. \tag{5.6}$$

Hence, if one chooses the gravitino mass to be of the order of $100\,\text{GeV}$, one finds μ and B of this order of magnitude, which, as we shall see somewhat later, may be consistent with radiative breaking of the electroweak symmetry.

We now proceed to a more detailed discussion of the simple but viable ORT models with and without the doublet-triplet splitting violating the SU(5) mass relations, studied in section 2 and 4. We shall be specially interested in phenomenology of the models with heavy messengers, i.e. with the values of X close to their upper bound.

In table 1 we collect a sample of models discussed in the previous sections; these models are characterized by the effective number of messengers. Cases A-C correspond to the usual models of gauge mediation with equal number of messengers in the doublet and triplet sectors. cases D and E are described in section 4.1, and cases F and G in section 4.2.

With fixed $N_{\text{eff},2}$ and $N_{\text{eff},3}$, the effective continuous parameters of these models of gauge mediation can be chosen, e.g., as the messenger mass scale $\mathcal{M}_{\text{mess}}$ and the gaugino (say, gluino) initial mass. We impose the boundary conditions for the soft masses (4.1) and (4.2) at the scale $\mathcal{M}_{\text{mess}}$. The two parameters are related to the gravitino mass by (5.2)
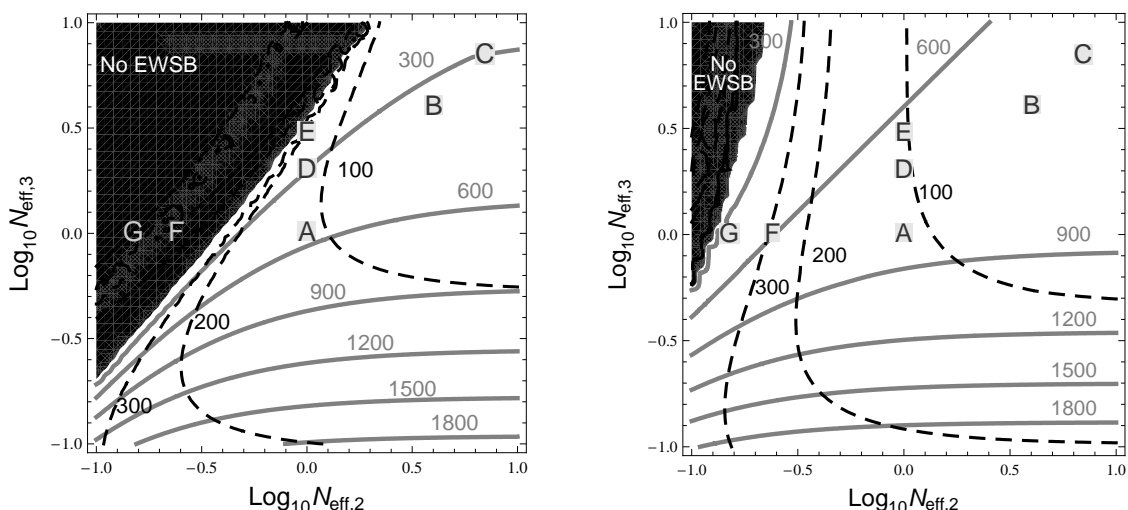
Figure 7: Values of μ (solid gray lines) and B (dashed black) in GeV as functions of the effective numbers of messengers $N_{\text{eff},2}$ and $N_{\text{eff},3}$ for two messenger scales $\mathcal{M}_{\text{mess}} = 2 \cdot 10^5 \, \text{GeV}$ (left panel) and $\mathcal{M}_{\text{mess}} = 2 \cdot 10^{15} \, \text{GeV}$ (right panel). Black regions correspond to the absence of electroweak symmetry breaking. Points corresponding to spectra A-G defined in table 1 are denoted by appropriate letters.

or, more explicitly:

$$m_{3/2} = \frac{4\pi}{\sqrt{3}\alpha} \frac{\mathcal{M}_{\text{mess}}}{M_P} \frac{m_{\text{gaugino}}}{\lambda_{\text{mess}}} \tag{5.7}$$

In the following, we take $F \, \partial_X \ln \det \mathcal{M} = 140 \, \text{TeV}$ in (4.1), which approximately corresponds to the gluino mass of $1 \, \text{TeV}$ at the electroweak scale, and vary the messenger mass between $\mathcal{M}_{\text{mess}} = 2 \cdot 10^5 \, \text{GeV}$ and $\mathcal{M}_{\text{mess}} = 2 \cdot 10^{15} \, \text{GeV}$. The lower limit comes from the requirement that $F/\bar{m}^2 < 1$, for which the effective Kähler potential technique applies, the upper limit follows from our results in section 3.1. For these values of the parameters, we plot in figure 6 $m_{3/2}$ as a function of $\mathcal{M}_{\text{mess}}$ for different values of λ_{mess}.

The requirement of proper electroweak symmetry breaking fixes the MSSM parameters μ and B in terms of the initial values of soft masses. In figure 7 we show the predictions for μ (solid gray lines, values in GeV) and for B (dashed black lines, values in GeV) as function of $N_{\text{eff},2}$ and $N_{\text{eff},2}$ for the two extremal values of the messenger masses, $\mathcal{M}_{\text{mess}} = 2 \cdot 10^5 \, \text{GeV}$ (left) and $\mathcal{M}_{\text{mess}} = 2 \cdot 10^{15} \, \text{GeV}$ (right). These values of μ and B are given at the scale $\mathcal{M}_{\text{mess}}$. The values of N_{eff} for the seven cases listed in table 1 are denoted by the corresponding letters. We chose $\tan\beta = 10$, different choices of this parameter do not affect μ much, while B is very sensitive to the value of $\tan\beta$ (this dependence is quantitatively discussed in appendix D). The results shown in figure 7 are obtained from numerical calculation based on 1(2)-loop RG equations for the dimension(less) parameters of the MSSM (with the exception of the $Y = +1/2$ higgs mass parameter, for which the 2-loop RGE was used). As we show in appendix D, they can be understood quite easily, e.g., with help of the analytical solutions to the RGE for nonuniversal soft masses given in [58].
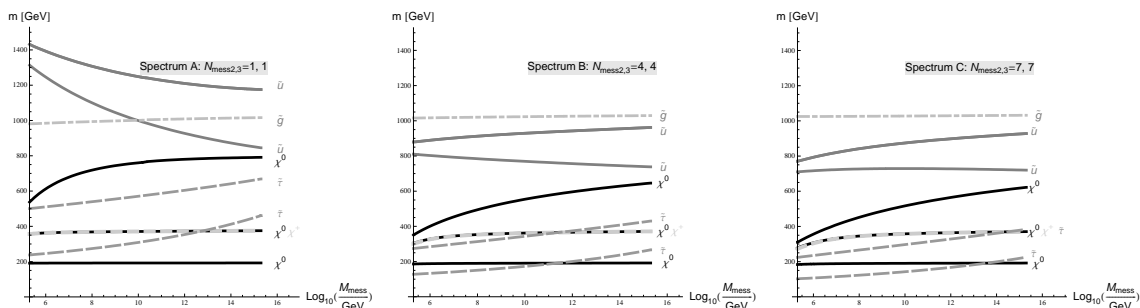
Figure 8: Masses of the three lightest neutralinos, the lightest chargino, the lightest left- and right-handed sleptons, the two lightest stops and the gluino as functions of the messenger scale for spectra A-C defined in table 1.
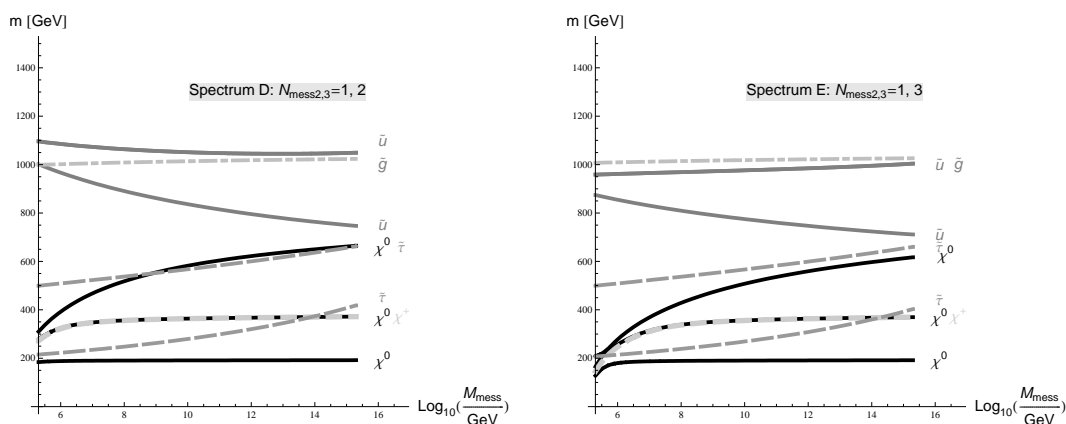

Figure 9: Masses of the three lightest neutralinos, the lightest chargino, the lightest left- and right-handed sleptons, the two lightest stops and the gluino as functions of the messenger scale for spectra D and E defined in table 1.

We note from figure 7 that for the same values of $N_{\text{eff},2}$ and $N_{\text{eff},3}$ the ratio μ/B increases with the messenger mass $\mathcal{M}_{\text{mess}}$. Furthermore, for $N_{\text{eff},2} \approx N_{\text{eff},3}$ and for $N_{\text{eff},2} > N_{\text{eff},3}$, $\mu/B \gg 1$ whereas for $N_{\text{eff},2} < N_{\text{eff},3}$ we have $\mu/B \approx 1$. This is interesting in case of heavy messengers (right panel in figure 7). As we discuss in some detail in appendix D, there exist then solutions with $\mu \sim B \approx m_{3/2} \approx O(100)\,\text{GeV}$, opening the possibility of generating μ and B by the Giudice-Masiero mechanism.

Our results for the supersymmetric spectra are shown in figures 8–10. There we show the masses of the three lightest neutralinos, the lightest chargino, the lightest left- and right-handed sleptons, the two lightest stops and the gluino. The dependence on the messenger masses is rather weak, except for cases F and G where a portion of the assumed messenger mass range does not lead to a correct electroweak symmetry breaking, as seen by comparing the two panels of figure 7. Generically, the NLSP is always bino-like neutralino. There are, however, two interesting exceptions. One is the stau as NLSP for large number of messengers (with $N_{eff,2} = N_{eff,3}$). This effect is present also for models with heavy
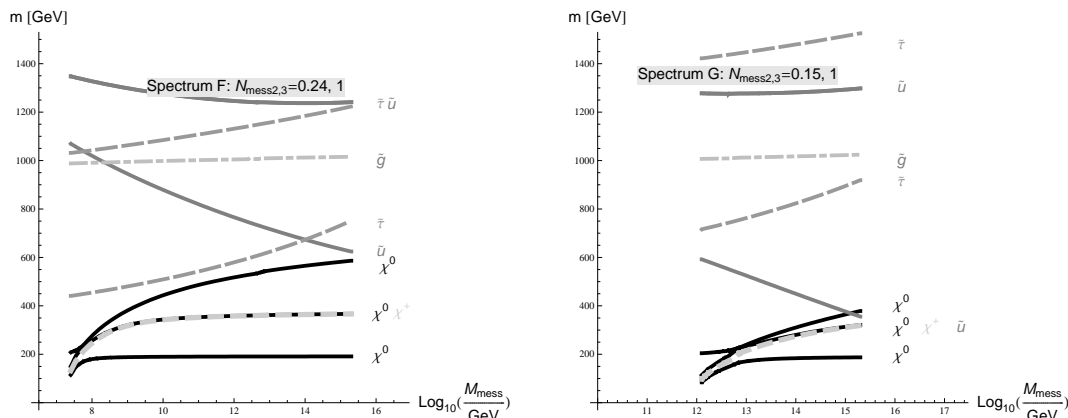
Figure 10: Masses of the three lightest neutralinos, the lightest chargino, the lightest left- and right-handed sleptons, the two lightest stops and the gluino as functions of the messenger scale for spectra F and G defined in table 1.

messengers, with gravitino mass even up to $100\,\text{GeV}$. The second exception is for models with $N_{eff,2} \ll N_{eff,3}$ where the lightest neutralino becomes higgsino-like if the scale $\mathcal{M}_{\text{mess}}$ is such that various contributions to the mass of the $Y = +1/2$ higgs approximately cancel. This regime with small μ and a large higgsino admixture in the lightest neutralino corresponds, in figure 10, to the situation in which the lines denoting the neutralino masses exhibit level crossing. As we show in section 4.2 and in appendix C, λ_{mess} can be as small as 10^{-3} (or even smaller). Thus, for gravitino masses in the range 10–$100\,\text{GeV}$ the higgsino-like neutralino as NLSP is also an open possibility.

6. Summary and conclusions

We have investigated O'Raifeartaigh-type models for F-term supersymmetry breaking in gauge mediation scenarios in the presence of gravity. In section 2 the vacuum structure of a broad class of models has been studied in the global supersymmetry limit. Gravity effects have been discussed in section 3. The gravity sector may include properly stabilized moduli. In section 4, we have discussed models with broken SU(5) symmetry in the messenger sector but equipped with messenger parity (and the disastrous consquences of the lack thereof).

The main conclusion is that, after coupling to gravity, the vacuum structure of those models is such that in metastable vacua gauge mediation is always dominant and gravity mediation contribution to scalar masses is suppressed to the level below 1 percent, almost sufficient for avoiding FCNC problem. Close to that limit, gravitino mass can be in the range 10–$100\,\text{GeV}$, opening several interesting possibilities for gauge mediation models, which are briefly discussed in section 5. One is that in models with broken SU(5) symmetry, the values of μ and B fixed by requiring the electroweak symmetry breaking include the region $\mu \approx B \approx m_{3/2} \sim O(100)\,\text{GeV}$, thus allowing for Giudice-Masiero mechanism for μ and $B\mu$ generation. We also show sparticle spectrum as a function of the gravitino mass. The NLSP is generically bino-like neutralino but exceptionally can be stau or higgsino.

Acknowledgments

A. Perturbative calculation of masses in the O'Raifeartaigh sector

Here we outline the perturbative procedure of calculating the eigenvalues of $\mathcal{M}^\dagger\mathcal{M}$. In the limit $|X| \to 0$, the eigenvalues are $\mu_0^{(k)} = m_k^2$ and the corresponding eigenvectors are ${}^0v_i^{(k)} = \delta_{ki}$. At the n-th level of the expansion in X, we find the coefficients of the $|X|^n$ contribution to the eigenvalues $\mu_n^{(k)}$ and the eigenvectors ${}^nv_i^{(k)}$ from the perturbed secular equation:

$$m_i^2 \left({}^nv_i^{(k)}\right) + \sum_{j=1}^{N}(m_j\lambda_{ji}^*\bar{X} + m_i\lambda_{ij}X)\left({}^{n-1}v_j^{(k)}\right) + \sum_{j,\ell=1}^{N}\lambda_{\ell i}^*\lambda_{\ell j}\bar{X}X\left({}^{n-2}v_j^{(k)}\right) =$$
$$= \sum_{\ell=0}^{n}\mu_\ell^{(k)}\left({}^{n-\ell}v_i^{(k)}\right) \tag{A.1}$$

assuming $\mu_n^{(k)} = {}^nv_i^{(k)} = 0$ for $n < 0$. The coefficients $\mu_n^{(k)}$ can be equivalently defined as:

$$\mu_{2j}^{(k)} = \frac{1}{j!}\frac{\bar{m}^{2j}}{\bar{\lambda}^{2j}}\left(\frac{\partial^2}{\partial\bar{X}\partial X}\right)^j \mathcal{M}_k^2 \Bigg|_{|X|=0}, \tag{A.2}$$

where $\mathcal{M}_k^2$ are the eigenvalues of $(\mathcal{M}^{\mathrm{ns}})^\dagger\mathcal{M}^{\mathrm{ns}}$, introduced in section 2. The first functions $f_{2\ell}$ are given by:

$$f_2 = \frac{1}{\bar{m}^2}\sum_{k=1}^{N}\left(\mu_2^{(k)}\ln\frac{m_k^2}{Q^2} + \mu_2^{(k)}\right) \tag{A.3}$$

$$f_4 = \frac{1}{\bar{m}^4}\sum_{k=1}^{N}\left(\mu_4^{(k)}\ln\frac{m_k^2}{\bar{m}^2} + \frac{\bar{m}^2}{2}\frac{(\mu_2^{(k)})^2}{m_k^2}\right) \tag{A.4}$$

$$f_6 = \frac{1}{\bar{m}^6}\sum_{k=1}^{N}\left(\mu_6^{(k)}\ln\frac{m_k^2}{\bar{m}^2} + \bar{m}^2\frac{\mu_2^{(k)}\mu_4^{(k)}}{m_k^2} - \frac{\bar{m}^4}{6}\frac{(\mu_2^{(k)})^3}{m_k^4}\right) \tag{A.5}$$

Using this procedure, we can express the coefficients $\mu_2^{(k)}$ and $\mu_4^{(k)}$ in terms of the original parameters of the model, the relevant general formulae are, however, rather lengthy. They significantly simplify under assumption that $R(\phi_1) > R(\phi_1) > \ldots > R(\phi_N)$ and $R(\tilde{\phi}_1) < R(\tilde{\phi}_2) < \ldots < R(\tilde{\phi}_N)$, i.e. the fields have different R charges, since the matrix of the couplings has then only a few nonzero entries, $\lambda_{ij} = \bar{\lambda}q_i e^{\iota\varphi_i}\delta_{i+1,j}$. We then obtain:

$$\frac{\mu_2^{(k)}}{\bar{m}^2} = \frac{\rho_k^2 q_{k-1}^2}{\rho_k^2 - \rho_{k-1}^2} + \frac{\rho_k^2 q_k^2}{\rho_k^2 - \rho_{k+1}^2} \tag{A.6}$$

$$\frac{\mu_4^{(k)}}{\bar{m}^2} = \frac{\rho_k^2\rho_{k-1}^2 q_{k-1}^2 q_{k-2}^2}{(\rho_k^2-\rho_{k-1}^2)^2(\rho_k^2-\rho_{k-2}^2)} + \frac{\rho_k^2\rho_{k+1}^2 q_k^2 q_{k+1}^2}{(\rho_k^2-\rho_{k+1}^2)^2(\rho_k^2-\rho_{k+2}^2)} -$$
$$-\frac{\rho_k^2(\rho_k^4-\rho_{k-1}^2\rho_{k+1}^2)q_k^2 q_{k-1}^2}{(\rho_k^2-\rho_{k-1}^2)^2(\rho_k^2-\rho_{k+1}^2)^2} - \frac{\rho_k^2\rho_{k-1}^2 q_{k-1}^4}{(\rho_k^2-\rho_{k-1}^2)^3} - \frac{\rho_k^2\rho_{k+1}^2 q_k^4}{(\rho_k^2-\rho_{k+1}^2)^3} \tag{A.7}$$

$$\frac{\mu_6^{(k)}}{\bar{m}^2} = \frac{\rho_k^2\rho_{k-1}^2\rho_{k-2}^2 q_{k-1}^2 q_{k-2}^2 q_{k-3}^2}{(\rho_k^2-\rho_{k-1}^2)^2(\rho_k^2-\rho_{k-2}^2)^2(\rho_k^2-\rho_{k-3}^2)} + \frac{\rho_k^4\rho_{k-1}^2 q_{k-1}^4 q_{k-2}^4}{(\rho_k^2-\rho_{k-1}^2)^3(\rho_k^2-\rho_{k-2}^2)^2} +$$
$$+\frac{\rho_k^2\rho_{k-1}^2(-2\rho_k^4+2\rho_{k-1}^2\rho_{k+1}^2+\rho_k^2\rho_{k-2}^2-\rho_k^2\rho_{k-1}^2)q_{k-1}^4 q_{k-2}^2}{(\rho_k^2-\rho_{k-1}^2)^4(\rho_k^2-\rho_{k-2}^2)^2} + \frac{\rho_k^2\rho_{k-1}^2(\rho_k^2+\rho_{k_1}^2)q_{k-1}^6}{(\rho_k^2-\rho_{k-1}^2)^5} -$$
$$-\frac{\rho_k^2\rho_{k-1}^2(3\rho_k^6+\rho_k^2\rho_{k-2}^2\rho_{k+1}^2+\rho_{k-1}^2\rho_{k-2}^2\rho_{k+1}^2-\rho_k^4(\rho_{k-1}^2+2\rho_{k-2}^2+2\rho_{k+1}^2))q_k q_{k-1}^2 q_{k-2}^2}{(\rho_k^2-\rho_{k-1}^2)^3(\rho_k^2-\rho_{k-2}^2)^2(\rho_k^2-\rho_{k+1}^2)} +$$
$$+\frac{\rho_k^2(\rho_k^8+2\rho_k^6\rho_{k-1}^2-6\rho_k^4\rho_{k-1}^2\rho_{k+1}^2+2\rho_k^2\rho_{k-1}^2\rho_{k+1}^4+\rho_{k-1}^4\rho_{k+1}^4)q_{k-1}^4 q_k^2}{(\rho_k^2-\rho_{k-1}^2)^4(\rho_k^2-\rho_{k+1}^2)^3} +$$
$$+\frac{\rho_k^2(\rho_k^8+2\rho_k^6\rho_{k+1}^2-6\rho_k^4\rho_{k-1}^2\rho_{k+1}^2+2\rho_k^2\rho_{k-1}^4\rho_{k+1}^2+\rho_{k-1}^4\rho_{k+1}^4)q_{k-1}^2 q_k^4}{(\rho_k^2-\rho_{k-1}^2)^3(\rho_k^2-\rho_{k+1}^2)^4} -$$
$$-\frac{\rho_k^2\rho_{k+1}^2(3\rho_k^6+\rho_k^2\rho_{k-1}^2\rho_{k+2}^2+\rho_{k-1}^2\rho_{k+1}^2\rho_{k+2}^2-\rho_k^4(\rho_{k+1}^2+2\rho_{k-1}^2+2\rho_{k+2}^2))q_{k+1}^2 q_k^2 q_{k-1}^2}{(\rho_k^2-\rho_{k-1}^2)^2(\rho_k^2-\rho_{k+1}^2)^3(\rho_k^2-\rho_{k+2}^2)} +$$
$$+\frac{\rho_k^4\rho_{k+1}^2 q_k^2 q_{k+1}^4}{(\rho_k^2-\rho_{k+1}^2)^3(\rho_k^2-\rho_{k+2}^2)^2} + \frac{\rho_k^2\rho_{k+1}^2(-2\rho_k^4+2\rho_{k+1}^2\rho_{k+2}^2+\rho_k^2\rho_{k+2}^2-\rho_k^2\rho_{k+1}^2)q_k^4 q_{k+1}^2}{(\rho_k^2-\rho_{k+1}^2)^4(\rho_k^2-\rho_{k+2}^2)^2} +$$
$$+\frac{\rho_k^2\rho_{k+1}^2\rho_{k+2}^2 q_k^2 q_{k+1}^2 q_{k+2}^2}{(\rho_k^2-\rho_{k+1}^2)^2(\rho_k^2-\rho_{k+3}^2)^2(\rho_k^2-\rho_{k+3}^2)} + \frac{\rho_k^2\rho_{k+1}^2(\rho_k^2+\rho_{k+1}^2)q_k^6}{(\rho_k^2-\rho_{k+1}^2)^5} \tag{A.8}$$

B. Simple examples of O'Raifeartaigh-type models

Here we present the results for the functions f_4 and f_6 calculated in some simple Type I models.

(i) With only $N=2$ pair of fields $\tilde{\phi}_i$ and ϕ_i there is only one coupling and one mass ratio ρ. One immediately obtains $f_4 = (1+\rho^2)/(2(\rho^2-1)^2) - \rho^2(\ln\rho^2)/(\rho^2-1)^3 > 0$.

(ii) Let us consider equally spaced masses squared, $\rho_k^2 = 1 + k\delta$. In the limit $\delta \to 0$ we obtain $f_4 = (1/12)\left[q_1^4 + q_N^4 + \sum_{k=1}^{N-1}(q_k^2 - q_{k+1}^2)^2\right] > 0$.

(iii) We assume 3 pairs of fields $\tilde{\phi}_i$ and ϕ_i with the most general mass and coupling structure, $\rho_1 = 1$, $\rho_2 = \rho$, $\rho_3 = \tilde{\rho}$, $q_1 = 1$ and $q_2 = q$, generalizing the model considered in [39]. We obtain

$$f_4 = \frac{1+\rho^2}{2(\rho^2-1)^2} + \frac{\rho^2 q}{(\rho^2-1)(\rho^2-\tilde{\rho}^2)} + \frac{(\rho^2+\tilde{\rho}^2)q^2}{2(\rho^2-\tilde{\rho}^2)^2} +$$
$$+ \left[\frac{\rho^2\tilde{\rho}^2 q}{(\tilde{\rho}^2-1)(\rho^2-\tilde{\rho}^2)^2} + \frac{\rho^2\tilde{\rho}^2 q^2}{(\rho^2-\tilde{\rho}^2)^3}\right]\ln\tilde{\rho}^2 -$$
$$- \left[\frac{\rho^2}{(\rho^2-1)^3} + \frac{\rho^2(\rho^4-\tilde{\rho}^2)q}{(\rho^2-1)^2(\rho^2-\tilde{\rho}^2)^2} + \frac{\rho^2\tilde{\rho}^2 q^2}{(\rho^2-\tilde{\rho}^2)^3}\right]\ln\rho^2 \tag{B.1}$$
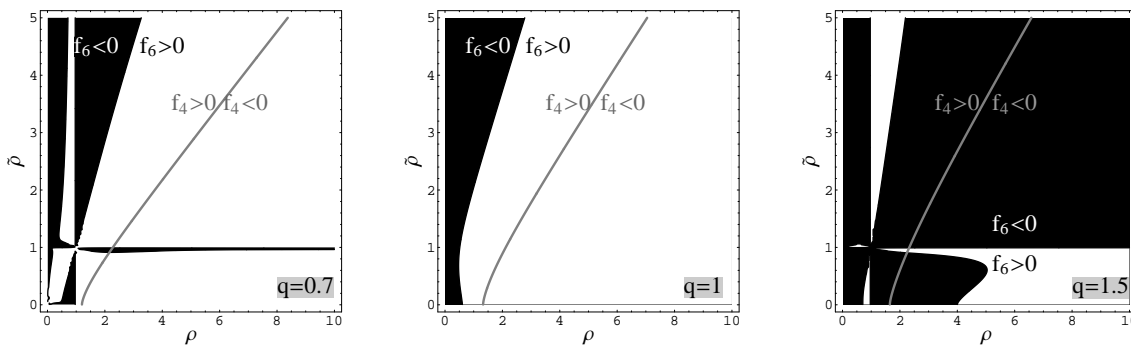
Figure 11: For the model discusses in Example (iii), we plot the sign of the coefficient f_4, given in (B.1), and the coefficient f_6 as functions of the mass ratios ρ and $\tilde{\rho}$ for three values of the model parameter q. The black (white) region corresponds to $f_6 < 0$ ($f_6 > 0$). Regions to the left (right) of the gray line correspond to $f_4 > 0$ ($f_4 < 0$).

The sign of the expression (B.1) is shown in figure 11 as a function of ρ and $\tilde{\rho}$.

(iv) We assume N pairs of fields $\tilde{\phi}_i$ and ϕ_i. We randomize their relative couplings q_k in the range $[1, q_{\max}]$ with a constant probability density for $\ln q_k$. We randomize ρ_k with a constant probability density for $\ln \rho_k$, rescaling the results so that the ratio between the smallest and the largest ρ_k is exactly $\rho_{\max}$. We repeat this procedure 100 times to estimate the probability of the event of interest (either $f_4 < 0$ and $f_6 > 0$ or $f_4 < 0$ and $f_6 < 0$). We perform this calculation 20 times to assess the variance of this prediction. Results showing the dependence $P(f_4 < 0 \text{ and } f_6 > 0)$ and $P(f_4 < 0 \text{ and } f_6 < 0)$ on $\rho_{\max}$ for $N = 4, 20$ are shown in figure 12 as light gray (black) dots for $q_{\max} = 1 \, (10)$. We can conclude, in concordance with Example (iii), that a supersymmetry breaking minimum with $X \neq 0$ exists in a significant fraction of the parameter space, given that the mass hierarchies are sufficiently large. This fraction is generically larger for degenerate than nondegenerate couplings.

Below we also discuss some examples of models in which the messenger sector can affect the position of the supersymmetry breaking minimum.

(v) In the absence of the O'Raifeartaigh sector and for $\tilde{N} = 2$, $\mathcal{M}^s$ is given by

$$\mathcal{M}^s = \begin{pmatrix} \lambda_1 X & m \\ 0 & \lambda_2 X \end{pmatrix} \tag{B.2}$$

and we can find the analytic form of eigenvalues of $\mathcal{M}^{s\dagger}\mathcal{M}^s$ and numerically solve for a minimum of the effective potential. In figure 13a we plot the value of $\sqrt{\lambda_1 \lambda_2} X/m$ at the minimum as a function of $\sqrt{\lambda_1/\lambda_2}$ (and the results agree with those from [31] for this parameter equal to 1).

(vi) For $\tilde{N} > 2$, the eigenvalues of $\mathcal{M}^{s\dagger}\mathcal{M}^s$ have to be computed numerically and f_4, f_0 are functions of three or more parameters. We may therefore employ the expansion (2.8)
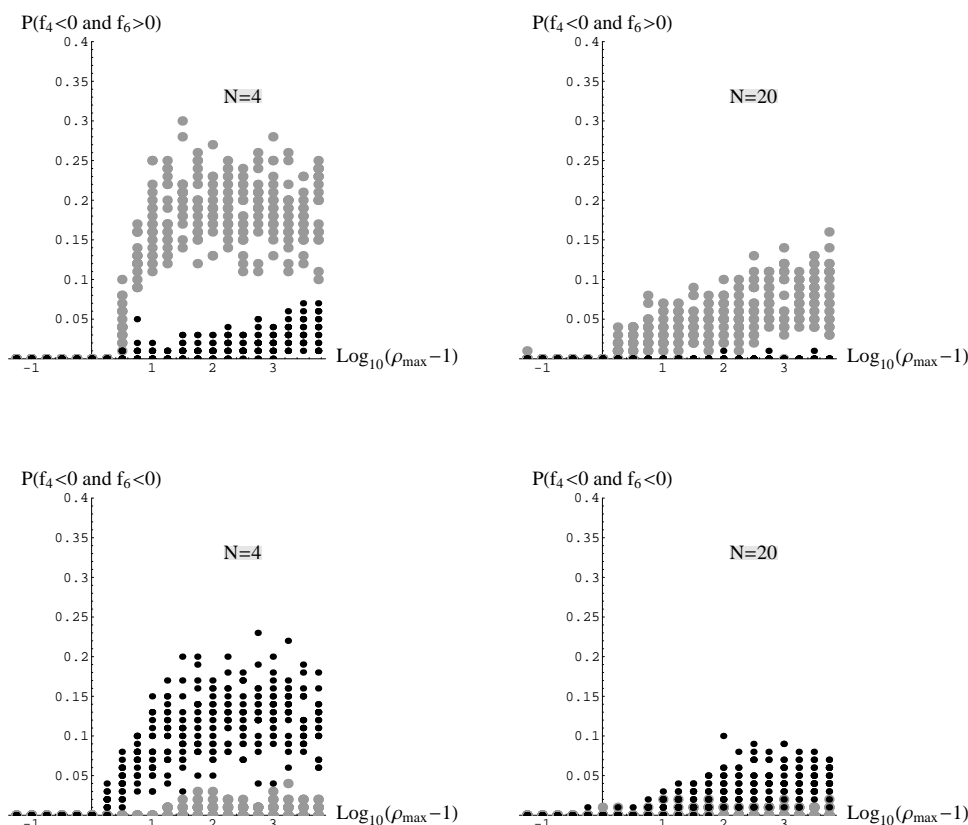
Figure 12: Results of the numerical procedure employed in Example (iv). The plots show $P(f_4 < 0 \text{ and } f_6 > 0)$ and $P(f_4 < 0 \text{ and } f_6 < 0)$ for different values of $\rho_{\max}$ for $N = 4$ and $N = 20$. Light gray (black) dots correspond to maximal ratio of couplings in the O'Raifeartaigh sector equal to $q_{\max} = 1\,(10)$.

to look for a supersymmetry breaking minimum even for $f_4 > 0$, taking f_4/f_0 as a free parameter and neglecting the higher order terms. The results for of $\bar{\lambda}|X|/\bar{m}$ at the minimum are shown in figure 13b. Of course, for large values of the expansion parameter $\bar{\lambda}|X|/\bar{m}$ higher order corrections can become important and one should treat these results only as qualitative estimates.

C. Calculation of the effective messenger numbers

Here we discuss corrections to the expression (4.4) for $N_{\text{eff},r}$ originating from messenger mass splittings.

In the Giudice-Rattazzi formalism [53], the soft masses of the scalars are extracted from the relation

$$m_{\tilde{f}}^2(\mu) = -|F|^2 \partial_{\bar{X}} \partial_X \ln Z_{\tilde{f}}(\mu, X, X^\dagger) \tag{C.1}$$
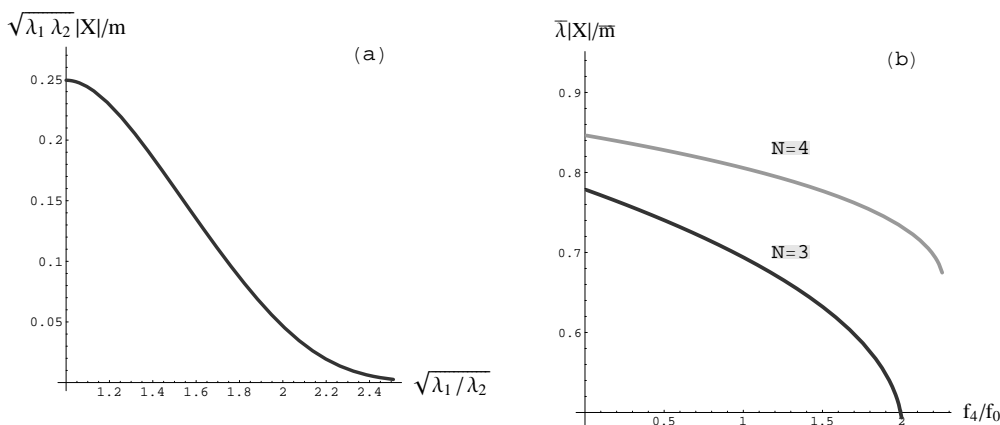
Figure 13: (a) The location for the supersymmetry breaking minimum calculated in the absence of supergravity corrections in Example (v); (b) the location for the supersymmetry breaking minimum calculated in the absence of supergravity corrections in Example (vi) for a truncated correction (2.8) to the Kähler potential.

where $Z_{\tilde{f}}$ is the sfermion wave function renormalization constant satisfying:

$$\frac{\mathrm{d} \ln Z_{\tilde{f}}}{\mathrm{d}\tau} = \sum_{r=1}^{3} C_{\tilde{f}}^{r} \frac{g_r^2}{4\pi^2} \,, \tag{C.2}$$

where $\tau = (1/(16\pi^2)) \ln(\mu/\Lambda_{\mathrm{UV}})$. Let us restrict the discussion to a single gauge group (dropping the index r from now on), as the contributions from different gauge groups to $\ln Z_{\tilde{f}}$ add up. Equation (C.2) can be rewritten using the RGE for the gauge coupling:

$$\frac{\mathrm{d} \ln Z_{\tilde{f}}}{\mathrm{d}\tau} = -\frac{2C_{\tilde{f}}}{\beta_g} \frac{\mathrm{d} \ln g^{-2}}{\mathrm{d}\tau} \,, \tag{C.3}$$

where β_g is the MSSM beta function of the gauge coupling. Taking into account multiple messenger thresholds, we arrive at the result:

$$\ln Z_{\tilde{f}}(\mu, X, X^{\dagger}) - \ln Z_{\tilde{f}}(\Lambda_{\mathrm{UV}}) = -2C_{\tilde{f}} \sum_{j=0}^{\tilde{N}} \frac{1}{\beta_g + jS} \left\{ \ln \left[g^{-2}(\mathcal{M}_j) \right] - \ln \left[g^{-2}(\mathcal{M}_{j+1}) \right] \right\} \,, \tag{C.4}$$

where S is the Dynkin index of the messenger representation ($S = 1$ for a $\mathbf{N} + \bar{\mathbf{N}}$ pair of SU(N)) and $\mathcal{M}_0$ and $\mathcal{M}_{\tilde{N}+1}$ stand for μ and Λ_{UV}, respectively. Eq. (C.4) is rather complicated for practical applications. One can try to simplify the analysis by combining contributions to sfermion masses originating at each messenger threshold $\mu = \mathcal{M}_j$. Choosing μ and Λ_{UV} infinitesimally close to the threshold, one can rewrite the first line of (C.4) as:

$$\ln Z_{\tilde{f}}(\mu, X, X^{\dagger}) - \ln Z_{\tilde{f}}(\Lambda_{\mathrm{UV}}) = -2C_{\tilde{f}} \left[\frac{1}{\beta_g} \ln \left(\frac{g^{-2}(\mu)}{g^{-2}(\mathcal{M}_j)} \right) + \frac{1}{\beta_g + S} \ln \left(\frac{g^{-2}(\mathcal{M}_j)}{g^{-2}(\Lambda_{\mathrm{UV}})} \right) \right] \tag{C.5}$$
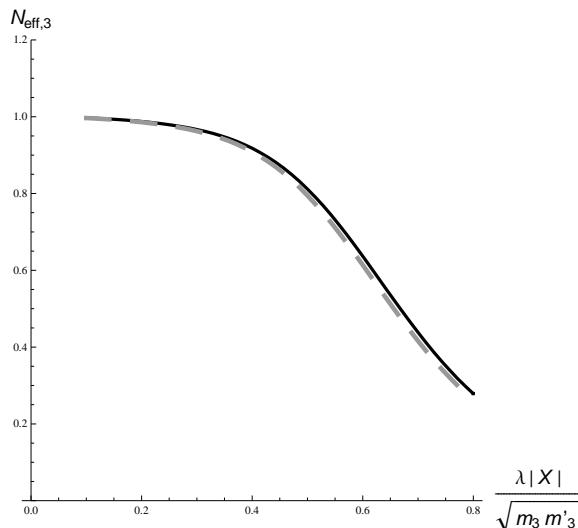
Figure 14: Values of N_{eff} calculated in the SU(5)-symmetric version of the model given in eq. (4.10) in section 4.2. The black solid line correspond to employing the full Giudice-Rattazzi method as described in appendix C; the gray dashed line correspond to the simplified result following from (4.2).

Expanding this expression in $\ln((\mathcal{M}_j)^2/\Lambda_{\text{UV}}^2)$, we find:

$$\ln Z_{\tilde{f}}(\mu, X, X^\dagger) - \ln Z_{\tilde{f}}(\Lambda_{\text{UV}}) = C_{\tilde{f}} S \left(\frac{g^2}{16\pi^2} \right)^2 \left(\ln \frac{\mathcal{M}_j^2}{\Lambda_{\text{UV}}^2} \right)^2 + O \left(\left(\ln \frac{\mathcal{M}_j^2}{\Lambda_{\text{UV}}^2} \right)^3 \right) \quad (C.6)$$

Summing all such results for $j = 1, \ldots, \tilde{N}$ and substituting to (C.1), we obtain the result (4.2).

In this derivation, we eventually ignored the higher powers of the logarithm, which is legitimate, as, after differentiation with respect to X and $X^\dagger$, we can set Λ_{UV} to the scale of the messenger threshold. One can, however, worry that in models with large splittings between the messenger masses, ignoring the running of the sfermion masses and gauge couplings between the messenger thresholds (which the full formula (C.4) takes into account up to top quark Yukawa corrections) can reduce the accuracy of the result (4.2). In order to address this issue quantitatively, we studied the model presented in eq. (4.11) in section 4.2 with $m_3/m_2 = 9$. For simplicity we took the triplet sector in which the mass splittings are the largest and, without stabilizing X at the minimum of the potential, we calculated $N_{\text{eff},3}$ using the full Giudice-Rattazzi result (C.4) and the simplified method following from (4.2). Since the gauge coupling had to be specified in the former approach, we took $g^2 = 2/3$ and $\beta_g = -3$. A comparison of the two calculations is presented in figure 14. We see that even with the messenger mass hierarchy larger than 10^3 in this model, the simplified result does not deviate from the more accurate one. A similar calculation with an even larger mass ratio $m_3/m_2 = 100$ revealed only small discrepancies of the order of 10% between the two approaches. We therefore conclude that for practical purposes we can safely neglect the threshold corrections to (4.2) coming from split messenger masses.

D. Semi-analytical analysis of radiative breaking of the electroweak symmetry in the MSSM

Here we employ the analytical formulae for the solutions of the 1-loop RGE for the MSSM derived in [58] to discuss the radiative breaking of the electroweak symmetry and the predictions for μ and B for different choices of the effective messenger numbers $N_{\text{eff},r}$.

The following equations describe the minimization conditions of the higgs sector of the scalar potential of the MSSM:

$$\mu^2 \simeq -m_{H_2}^2(\tau) \tag{D.1}$$

$$B(\tau)\mu \tan\beta \simeq m_{H_1}^2(\tau) - m_{H_2}^2(\tau) \tag{D.2}$$

The running mass parameters are presented as functions of $\tau = \frac{1}{(4\pi)^2}\ln\frac{M_{\text{low}}}{\mathcal{M}_{\text{mess}}}$. Here we neglected the running of μ, terms proportional to M_Z^2 and $1/\tan^2\beta$. We also assumed that the scale M_{low} at which (D.1) and (D.2) are evaluated is chosen so that the threshold y_t^4-proportional corrections to the masses and couplings relevant for EWSB vanish, i.e. M_{low} is the geometric mean of the stop masses.

The parameters $m_{H_2}^2(\tau)$, $m_{H_1}^2(\tau)$ and $B(\tau)$ can be expressed in terms of the input values $m_{H_2}^2(0)$, $m_{H_1}^2(0)$ and $B(0)$ at the scale $\mathcal{M}_{\text{mess}}$:

$$m_{H_2}^2(\tau) = \left(1 - \frac{y}{2}\right)m_{H_2}^2(0) - \frac{y}{2}\left(m_Q^2(0) + m_U^2(0)\right) + \left(\eta_{H_2} + \frac{y}{2}(-\hat\eta + y\hat\xi^2)\right)M_{1/2}^2 +$$
$$+ \frac{1}{22}\left(\frac{g_1^2(\tau)}{g_1^2(0)} - 1\right)\left(m_{H_2}^2(0) - m_{H_1}^2(0)\right) \tag{D.3}$$

$$m_{H_1}^2(\tau) = m_{H_1}^2(0) + \eta_{H_2}M_{1/2}^2 - \frac{1}{22}\left(\frac{g_1^2(\tau)}{g_1^2(0)} - 1\right)\left(m_{H_2}^2(0) - m_{H_1}^2(0)\right) \tag{D.4}$$

$$B(\tau) = B(0) + \left(-\xi_B + \frac{y}{2}\hat\xi\right)M_{1/2}. \tag{D.5}$$

Here, $y = y_t^2/(y_t^{\text{FP}})^2$, where y_t is the running top Yukawa coupling and y_t^{FP} is its quasi-fixed point value. $M_{1/2}$ is the gluino mass at $\mathcal{M}_{\text{mess}}$. The coefficients ξ_B, η_{H_2}, $\hat\xi$ and $\hat\eta$ in (D.3)–(D.5) depend on the gauge couplings, gaugino mass ratios and some group-theoretical factors; they are defined in [58]. In table 2, we present their values for two choices of $\mathcal{M}_{\text{mess}} = 2 \cdot 10^{15}\,\text{GeV}$ and $\mathcal{M}_{\text{mess}} = 2 \cdot 10^5\,\text{GeV}$, discussed in section 5. Since their values of these coefficients depend on threshold corrections to the gauge couplings (mainly to the strong coupling, $\hat\alpha_s = \alpha_s/(1 - \Delta\alpha_s)$) and the value of M_{low} at which we stop the RG evolution to study the electroweak symmetry breaking and to calculate the mass spectra of the supersymmetric particles, we show the results for $\alpha_s = 0.118$ and for different values of $\Delta\alpha_s$ and M_{low}.

Now we would like to use the solutions (D.3)–(D.5) to determine which classes of the initial conditions at the scale $\mathcal{M}_{\text{mess}}$ correspond to successful breaking of the electroweak symmetry with small μ and $B(0)$. To this end, we define $\Xi' = \mu^2/M_{1/2}^2$ and rewrite (D.1) as

$$\Xi_{H_2} - \Xi' = \frac{1}{N_{\text{eff},2}}\left[\frac{3}{2}(1-y)\frac{g_2^4(0)}{g_3^4(0)} + \left(\frac{9}{50} - \frac{13}{50}y\right)\frac{g_1^4(0)}{g_3^4(0)}\right] +$$

		$\mathcal{M}_{\mathrm{mess}} = 2 \times 10^{15}\,\mathrm{GeV}$			$\mathcal{M}_{\mathrm{mess}} = 2 \times 10^{5}\,\mathrm{GeV}$		
	$M_{\mathrm{low}} =$	M_Z	$500\,\mathrm{GeV}$	$1000\,\mathrm{GeV}$	M_Z	$500\,\mathrm{GeV}$	$1000\,\mathrm{GeV}$
$\Delta\alpha_3 = 0$	$(y_t^{\mathrm{FP}})^2 =$	1.30	1.26	1.25	2.52	2.94	3.21
	$\xi_B =$	0.508	0.485	0.476	0.056	0.044	0.039
	$\eta_{H_2} =$	0.427	0.409	0.402	0.023	0.018	0.016
	$\hat{\xi} =$	2.01	1.76	1.67	0.423	0.302	0.258
	$\hat{\eta} =$	11.3	9.00	8.25	1.21	0.778	0.693
$\Delta\alpha_3 = -0.10$	$(y_t^{\mathrm{FP}})^2 =$	1.24	1.21	1.20	2.46	2.89	3.16
	$\xi_B =$	0.527	0.503	0.493	0.060	0.047	0.042
	$\eta_{H_2} =$	0.459	0.440	0.432	0.026	0.021	0.018
	$\hat{\xi} =$	1.88	1.68	1.58	0.390	0.280	0.241
	$\hat{\eta} =$	9.92	8.07	7.43	1.08	0.705	0.583
$\Delta\alpha_3 = -0.20$	$(y_t^{\mathrm{FP}})^2 =$	1.19	1.17	1.16	2.41	2.85	3.12
	$\xi_B =$	0.545	0.521	0.511	0.064	0.050	0.045
	$\eta_{H_2} =$	0.491	0.471	0.463	0.030	0.024	0.021
	$\hat{\xi} =$	1.77	1.57	1.50	0.363	0.263	0.226
	$\hat{\eta} =$	8.89	7.33	6.79	0.974	0.647	0.537

Table 2: Coefficients entering the solutions (D.3)–(D.5) of the RG equations.

$$+ \frac{1}{N_{\mathrm{eff},3}} \left[-\frac{8}{3}y + \left(\frac{3}{25} - \frac{13}{75}y \right) \frac{g_1^4(0)}{g_3^4(0)} \right], \tag{D.6}$$

where $\Xi_{H_2} = -\eta_{H_2} + \frac{y}{2}(\hat{\eta} - y\hat{\xi}^2)$. This equation simplifies considerably if $\mathcal{M}_{\mathrm{mess}}$ is close to the unification scale, since we can assume that the gauge couplings are approximately equal:

$$\Xi_{H_2} - \Xi' = \frac{1}{N_{\mathrm{eff},2}} \left(\frac{42}{25} - \frac{101}{50}y \right) + \frac{1}{N_{\mathrm{eff},3}} \left(-\frac{8}{3} + \frac{19}{50}y \right). \tag{D.7}$$

The left-hand side of this equation is positive and of the order of a few. The coefficient of $1/N_{\mathrm{eff},2}$ is small and positive, while the coefficient of $1/N_{\mathrm{eff},3}$ is negative and of the order of unity. Thus, we can conclude that solutions with small $\mu \ll M_{1/2}$, which correspond to $0 < \Xi' \ll 1$, will require a small $N_{\mathrm{eff},2} \ll N_{\mathrm{eff},3}$. Eq. (D.6) also simplifies for $\mathcal{M}_{\mathrm{mess}}$ much smaller than the unification scale, when we can neglect the terms proportional to $g_1^4(0)$ and take $1 - y \approx 1$. We then obtain

$$\Xi_{H_2} - \Xi' = \frac{1}{N_{\mathrm{eff},2}} \frac{3}{2} \frac{g_2^4(0)}{g_3^4(0)} - \frac{1}{N_{\mathrm{eff},3}} \frac{8}{3}y. \tag{D.8}$$

Taking $\mathcal{M}_{\mathrm{mess}} = 2 \times 10^5$ as a particular example, we see that Ξ_{H_2} approximately vanishes, so the two terms on the right-hand side must cancel out:

$$N_{\mathrm{eff},3} \simeq N_{\mathrm{eff},2} \frac{16y}{9} \frac{g_2^4(0)}{g_3^4(0)} \tag{D.9}$$

The numerical coefficient is $\approx 3 \approx 10^{0.4}$, so this equation is a reasonable approximation of the results in figure 7.

We see that in both examples discussed above, one can obtain solutions with small μ, given that there is an appropriate hierarchy $N_{\text{eff},2} \ll N_{\text{eff},3}$. Due to this requirement, there are not any solutions corresponding to $N_{\text{eff},2} = N_{\text{eff},3}$ which is the outcome of conventional models with gauge mediated supersymmetry breaking.

Having found the conditions for the existence of the solutions with small μ, we would like to know if some of these solutions correspond to small $B^{(0)} = \pm\mu/\sqrt{3}$, as in (5.5) and (5.6). In order to determine it, we can rewrite (D.2) as:

$$\left(\pm\frac{1}{\sqrt{3}}\Xi' + \Xi_B \right) \Xi' \tan\beta \simeq \nu \,, \tag{D.10}$$

where

$$\nu = \frac{1}{N_{\text{eff},2}} \left(\frac{3}{2}\frac{g_2^4(0)}{g_3^4(0)} + \frac{9}{50}\frac{g_1^4(0)}{g_3^4(0)} \right) + \frac{1}{N_{\text{eff},3}}\frac{3}{25}\frac{g_1^4(0)}{g_3^4(0)} + \eta_{H_2} \tag{D.11}$$

and $\Xi_B = -\xi_B + y\hat{\xi}/2$. The solutions with small μ correspond to $N_{\text{eff},2} \ll N_{\text{eff},3}$, so we can write $\nu \approx \gamma/N_{\text{eff},2} + \eta_{H_2} \gtrsim O(1)$ with the coefficient γ changing from $3/2$ to $27/25$ for $\mathcal{M}_{\text{mess}}$ inreasing from $2 \cdot 10^5$ GeV to $2 \cdot 10^{15}$ GeV. Equation (D.10) has a solution with $0 < \Xi' \ll 1$, if $\nu/\tan\beta$ is a sufficiently small number; this can be obtained by choosing an appropriate value of $\tan\beta$, unless $N_{\text{eff},2}$ is very small.

To summarize, we can obtain solutions with small $B = |\mu|/\sqrt{3} \ll M_{1/2}$ for both low and high scale of the gauge mediated symmetry breaking, given that appropriate hierarchy between $N_{\text{eff},2}$ and $N_{\text{eff},3} \gg N_{\text{eff},2}$ is arranged for and that $N_{\text{eff},2}$ is not too small. These solutions require doublet-triplet splitting in the messenger sector; a much stronger splitting is necessary for solutions with a high scale $\mathcal{M}_{\text{mess}}$ for which the Giudice-Masiero mechanism can be made work.

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